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INCIPIENT REGIMES OF DRILL BIT WHIRLINGS ON UNEVEN BOTTOMS OF DEEP BORE-HOLES

Поставлена задача про коливання кружляння бурового долота, що обертається, в умовах його контактної взаємодії із шорсткуватим дном свердловини. Оскільки такий тип вібрацій зустрічається в 40% випадків буріння глибоких свердловин, проблема їх вивчення представляє як науковий, так і прикладний інтерес.

Встановлено, що ефект самозбурення таких коливань пов'язаний зі збільшенням сили контактного тиску між долотом та свердловиною і її наближенням до критичного значення Ейлера. У цих умовах згинна жорсткість бурильної колони зменшується й вона може випучувати. При цьому вісь долота нахиляється й починає прецесувати. В результаті долото втрачає здатність ковзати по дну й починає котитися по його поверхні, відстаючи або випереджаючи обертання бурильної колони. При такому русі точка контакту долота із зазначеною поверхнею (миттєвий центр швидкостей) може описувати надзвичайно складні траєкторії у вигляді петель і точок загострення й змінювати напрямок руху. В якості прикладу розглянуті коливання кружляння доліт сферичної форми.

Ключові слова: глибоке буріння, долото, неголономна динаміка, прямі та зворотні кружляння.

Поставлена задача о колебаниях кружения вращающегося бурового долота в условиях его контактного взаимодействия с шероховатым дном скважины. Поскольку такой тип вибраций встречается в 40% случаев бурения глубоких скважин, проблема их изучения представляет как научный, так и прикладной интерес.

Установлено, что эффект самовозбуждения таких колебаний связан с увеличением силы контактного давления между долотом и скважиной и ее приближением к критическому значению Эйлера. В этих условиях изгибная жесткость бурильной колонны уменьшается и она может выпучивать. При этом ось долота наклоняется и начинает прецессировать. В результате долото теряет способность скользить по дну и начинает катиться по его поверхности, отставая или опережая вращение бурильной колонны. При таком движении точка контакта долота с указанной поверхностью (мгновенный центр скоростей) может описывать чрезвычайно сложные траектории в виде петель и точек заострения и изменять направление движения. В качестве примера рассмотрены колебания кружения долот сферической формы.

Ключевые слова: глубокое бурение, долото, неголономная динамика, прямое и обратное кружение.

This paper studies the problem of whirl vibrations of rotating drill string bit under conditions of its contact interaction with the bore-hole bottom surface. The destructive effects of whirl vibrations of drill bits and drill strings have long been recognized. The backward whirling is believed to randomly happen to some degree in 40 % of all wells.

It is shown that the situation is essentially associated with enlargement of the compressive thrust force and its approaching to the Eulerian critical value. Then, the bending stiffness of the drill string reduces, it hogs and the diamond impregnations existing in the bit surface penetrate into the rock medium. As this takes place, the bit axis tilts and nutates. It should be particularly emphasized that at this stage, the bit is under action of the vertical force (weight on bit) which essentially exceeds all other forces. It presses the bit to the hole bottom (not to the hole wall) and the bit loses its ability to slide on the bore-hole bottom and begins to roll on its surface, lagging behind or outstripping the drill string rotation. As a result, the point of the bit contact with the reference surface (the instantaneous centre of velocities) may describe extremely complicated trajectories with loops and cuspidal points and to change the motion directions. As examples, whirling of spherical bit is considered.

Keywords: deep drilling, drill bits, nonholonomic dynamics, forward and backward whirls.

Introduction

In the majority of cases, previous studies of drill bit whirl have concentrated primarily on assumption that in whirling the rotating drill bit is compressed to the bore-hole wall and is rolling on its surface [1-3]. But in our opinion, a prominent phase of this effect proceeds at the initial stage of its generation. These phenomena occur when the drilling bit changes over from being in sliding contact to rolling contact with the bore-hole bottom and then instantaneous center of its rotation begins to move around the DS axis on the bore-hole bottom surface [4]. Depending on the rotative movement direction, the forward and backward whirls are distinguished [2, 5]. As this takes place, the backward whirl can achieve frequencies somewhere in the range from 5 to 30 times the speed of the DS rotation [4]. This speed makes the generated force effect especially detrimental, producing a set of high fatigue-loading and strong impact-loading conditions that can be responsible for intensive bending vibration and irremediable detriment to downhole tools when the bit does not

achieve the well wall, does not touch it, and is rolling on the well bottom.

In this case, the most influential axial force can be resolved into a vertical force or weight on bit (WOB) and a near-zero transverse component called the bit imbalance force. This force is usually referred to as a percentage of the WOB. Numerous measurements of the bit imbalanced evidence that commercial bits might be 2% imbalanced, nevertheless, 10% imbalanced is more typical. Hence, it can be concluded that the bit is compressed to the well bottom by the WOB force, while only 10% of its value are responsible for the tendency to compress it to the well wall. Therefore, it is more reasonable to associate these vibrations with the bit rolling on the bottom surface rather than on the well, as is traditionally considered in theoretical researches.

According to field observation, the main factor influencing whirl is the bit geometry. In papers [3, 5], the phenomena of the bits spinning and rolling on the well bottoms are simulated with the use of nonlinearfrictional and

nonholonomic models for bits with different shapes. As an example, spherical bit whirling on spherical bore-hole bottom is numerically studied. It permitted to find three types of its stable and unstable motions associated with forward and backward rolling as well as pure spinning.

The problem on whirling of rigid ellipsoidal bits on well bottoms is considered in [4]. The oblong and oblate ellipsoidal shapes are selected for investigation. Two mathematic models of the bit whirling founded on supposition of the capability of their pure rolling whirl and rolling with sliding are proposed. The unsteady regimes of their rolling initiated by their initial perturbations are analyzed through computer simulations. The oblate bits are shown to be more disturbance-sensitive and curves of their whirling trajectories have ansiform outlines with sharp turns. Summarizing, it may be noted that the effects considered in these papers are simulated on the basis of nonlinear statements. They are multi-parametric as depend on elastic, dynamic, geometric, kinematic, frictional, and structural characteristics of the system. Therefore, the particular cases treated by computer simulations are only intriguing and inviting attention to their beauty. To establish general regularities of these processes proceeding, the object-oriented analytic studies should be performed. They cannot be performed through the application of created non-linear mathematic models and should be oriented to incipient stage of the process evolvement which can be described by linearized equations.

1 Whirling of a spheric bit on spheric bottom surface

To elaborate mathematic model of a drill bit whirling, take into account that this vibration process is attended with elastic bending of the DS tube adjacent to its bottom hole assembly (BHA). Assume that it is possible to separate mentally the lower sections AB and BC located between two lower centralizers A and B, and the collar boundary C with the bit at its end (Fig. 1). In the model constructing, the feature of fundamental importance is simulation of the bit motion which can be dynamic or kinematic. To perform the choice, it is necessary to compare frequencies of free vibrations of the separated piece of the DS and angular velocity of the bit. For example, the first frequency of free vibrations of the steel three-point beam ABC with lengths $e = 2$ m, $l = 9$ m (Fig.1) and cross-section diameters $d_1 = 0,18$ m, $d_2 = 0,18$ m equals $f_1 = 106,7\ s^{-1}$, while the DS angular velocity may achieve only $\omega \approx 10-20\ s^{-1}$. Besides, the bit represents a hollow-centered body with small inertia moment. So, the considered whirlings represent quasi-static processes of the DS deforming, inertia forces can be neglected at its analysis, and the whirling motions can be reckoned as kinematic, proceeding with velocities determined by angular velocity ω of the DS rotation. Therefore, in the whirling study only elastic and constraint forces will be taken into account and kinematic stimulation of the system motion will be analyzed.

2 Model of a spherical bit rolling on a spherical bottom surface

Assume that a DS is elastically deflected from vertical and its spherical bit, rotating with angular velocity $\boldsymbol{\omega}$ around its axis, begins to roll on uneven spherical surface of the well bottom π (Fig. 2). Then, depending on the position of contact

point G with respect to the D point, lying in the rotation axis, the bit center C can rotate in the direction of the bit rotation (forward whirling – Fig. 2, a) or, every so often, in the opposite direction (backward whirling – Fig. 2, b). Introduce immovable coordinate system OXYZ and reference frame Oxyz fixed in the beam of the rotating DS. Vertical axes OZ and Oz coincide. As is traditional in nonholonomic mechanics [5], it is conveniently to examine this motion relative to its instantaneous center G of total rotation, because in this case the unknown constraint reaction **R** is excluded from the constitutive equations and only elastic axial (**T**) and shear (**Q**) forces and bending moment (**M**) participate in the force equilibrium. Let the radii of the bit and hole bottom surfaces be a and b, correspondingly.
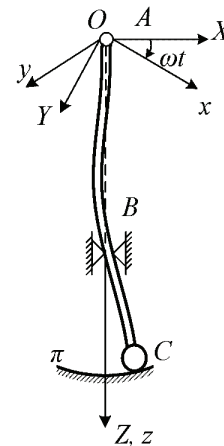

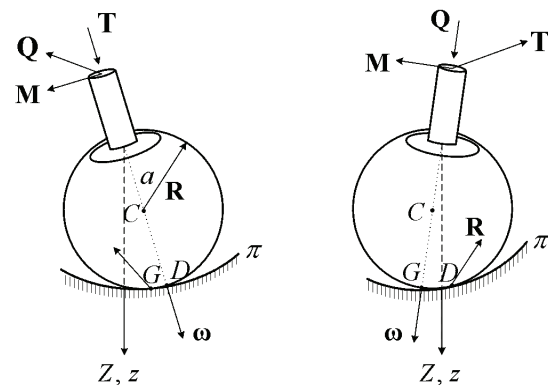
Figure 1 – Schematic of lower sections of a drill string

Figure 2 – Nonholonomic rolling of a spherical bit: forward whirling; b) backward whirling

If to assume that the contacting surfaces are absolutely uneven, then the sliding effects becomes impossible. In this case, $\mathbf{v} = 0$ and pure rolling with spinning is realized. Such model belongs to the nonholonomic type.

It is shown in paper [4] that the possibility of bit sliding the bore-hole bottom is severely reduced through the presence of diamond thorns on its surface. This effects is amplified by enlargement of the bit pressure on the hole bottom and decrease of the DS stiffness. Then, the nonholonomic model application is thoroughly justified.

In paper [3], nonlinear kinematic and dynamic equations were constructed, stemming from conditions of the spherical drill bit rolling without sliding and equilibrium of elastic moment **M** and shear force **Q** applied to the bit

(Fig. 2). They are based on the use of classical mechanics methods [6] and the theory of elastic beams [7, 8].

A special feature of this system of four equations lies in the fact that it contains eight unknown values u, u', u'', u''', and v, v', v'', v''' prescribed at the C edge of the DS beam. But this impediment can be surmounted if to take into account that the considered problem is quasi-static and consequently displacements u, v and turn angles u', v' can be expressed via bending moments $M_u = EIu''$, $M_v = EIv''$ and shear forces $Q_u = EIu'''$, $Q_v = EIv'''$ applied at this point. Then, the following equalities are valid:

$$u = w_M \cdot M_u + w_Q \cdot Q_u = w_M EIu'' + w_Q EIu''' ;$$
$$v = w_M \cdot M_v + w_Q \cdot Q_v = w_M EIv'' + w_Q EIv''' ;$$
$$u' = w_M^{(1)} \cdot M_u + w_Q^{(1)} \cdot Q_u = w_M^{(1)} EIu'' + w_Q^{(1)} EIu''' ;$$
$$v' = w_M^{(1)} \cdot M_v + w_Q^{(1)} \cdot Q_v = w_M^{(1)} EIv'' + w_Q^{(1)} EIv''' ,$$

where w_M is the elastic displacement of free edge C of the console beam shown in Fig.1 under action of unit bending moment $M = 1$ applied at this point, w_Q is the similar displacement resulted under action of unit shear force $Q = 1$ applied at the same point, $w_M^{()}$ is the elastic turn angle of the beam caused by the unit moment and $w_Q^{()}$ is the angle produced by the unit shear force.

With the aid of Eqs. (4), the next equalities can be gained

$$u'' = \frac{w_Q^{(1)}u - w_Q u'}{EI\left(w_M w_Q^{(1)} - w_Q w_M^{(1)}\right)} ;$$
$$u''' = \frac{w_M u' - w_M^{(1)}u}{EI\left(w_M w_Q' - w_Q w_M'\right)} ;$$
$$v'' = \frac{w_Q^{(1)}v - w_Q v'}{EI\left(w_M' w_Q^{(1)} - w_Q w_M^{(1)}\right)} ;$$
$$v''' = \frac{w_M v' - w_M^{(1)}v}{EI\left(w_M w_Q' - w_Q w_M^{(1)}\right)} ,$$

In this paper, small whirling vibrations are studied. Therefore, the nonlinear equations can be linearized and represented in the form:

$$\dot{u} + a\dot{u}' - \omega\left(1 + \frac{a}{b-a}\right)v + a\omega v' = 0 ;$$

$$\dot{v} + a\dot{v}' + \omega\left(1 + \frac{a}{b-a}\right)u - a\omega u' = 0 ;$$

$$\left[g^{(1)} - ah^{(1)} - \frac{Ta}{EI(b-a)}\right]u + \left(-g + ah + \frac{Ta}{EI}\right)u' = 0 ; \quad (1)$$

$$\left[g^{(1)} - ah^{(1)} - \frac{Ta}{EI(b-a)}\right]v + \left(-g + ah + \frac{Ta}{EI}\right)v' = 0 .$$

Here, the notations

$$D = EI\left(w_M w_Q^{(1)} - w_Q w_M^{(1)}\right); \quad \frac{w_M}{D} = h ; \quad \frac{w_M^{(1)}}{D} = h^{(1)} ;$$

$$\frac{w_Q}{D} = g ; \quad \frac{w_Q^{(1)}}{D} = g^{(1)} \qquad (2)$$

are introduced.

This system of four equations with four desired variables u, u', v and v' can receive further simplification through exclusion of variables u' and v'. Indeed, two last equations in this system permits one to write

$$u' = qu , \qquad v' = qv . \qquad (3)$$

Substituting Eqs. (3) into two first Eqs. (1), one gains the ultimate form of this system

$$\dot{u} - \omega pv = 0 , \qquad \dot{v} + \omega pu = 0 . \qquad (4)$$

Here,

$$p = \left(1 + \frac{a}{b-a} - aq\right) \Big/ (1 + aq). \qquad (5)$$

System (4) is equivalent to equations

$$\ddot{u} + \omega^2 p^2 u = 0 ; \qquad \ddot{v} + \omega^2 p^2 v = 0 . \qquad (6)$$

They have solutions

$$u(t) = C_1 \sin \omega pt + C_2 \cos \omega pt ,$$
$$v(t) = C_3 \sin \omega pt + C_4 \cos \omega pt . \qquad (7)$$

Here, constants C_1, C_2, C_3 and C_4 are determined from initial conditions.

They describe trajectories of the C center movement in the *Oxyz* coordinate system rotating with angular velocity **ω**. Denote $\omega p = \Omega_1$. Then, these correlations can be represented as follows:

$$u(t) = C_1 \sin \Omega_1 t + C_2 \cos \Omega_1 t ,$$
$$v(t) = -C_2 \sin \Omega_1 t + C_1 \cos \Omega_1 t . \qquad (8)$$

Here, Ω_1 is the whirling frequency in the rotating coordinate system.

3 Basic peculiarities of the whirl process

Though movement of a drill bit in rotating reference frame *Oxyz* is governed by simple Eqs.(6), it can assume large variety of modes, depending on initial conditions (constants C_1 and C_2) and value of parameter p which, in its turn, is specified by geometric values a and b, DS stiffness EI, and flexibility parameters $w_M, w_Q, w_M^{(1)}$, and $w_Q^{(1)}$. But it can outline much more complicated paths in fixed coordinate system *OXYZ*. These trajectories are traced with the use of equalities

$$X(t) = u(t)\cos\omega t - v(t)\sin\omega t ;$$
$$Y(t) = u(t)\sin\omega t + v(t)\cos\omega t . \qquad (9)$$

They testify that in the fixed reference frame, the bit moves with angular velocity $\Omega_2 = \omega(p - 1)$ and its motion is determined by equations

$$X(t) = C_1 \sin \Omega_2 t + C_2 \cos \Omega_2 t ;$$
$$Y(t) = -C_2 \sin \Omega_2 t + C_1 \cos \Omega_2 t . \qquad (10)$$

Eqs.(8) and Eqs.(10) permit one to analyze the possible tendencies for the spherical bit to perform different self-triggered whirlings depending on the frequency p values. Thus, if $p < -1$, then, as Eqs. (8) and (9) testify, the bit center C moves along circular trajectory with angular velocity $\Omega_1 > \omega$ in rotating reference frame *Oxyz* and performs forward whirling with angular velocity $\Omega_2 > 2\omega$, outstripping the DS rotation in fixed coordinate system *OXYZ*. Value $p = -1$ is in line with $\Omega_1 = \omega$, $\Omega_2 = 2\omega$.

Special regime is generated at $p = 0$. In this situation, the bit is motionless in rotating reference frame ($\Omega_1 = 0$) but performs forward whirling with frequency $\Omega_2 = \omega$ in

still space. The opposite situation occurs at $p = 1$ when the bit rotates in reverse direction with frequency $\Omega_1 = \omega$, though comes to a standstill at point $X = C_2$, $Y = C_1$ in the still space. Accentuate that this state is also unfavorable because here the bit performs pure spinning without rolling and sliding and the drilling direction begins to change its direction.

The subsequent enlargement of the parameter entails change-over from forward whirling of the bit to backward motion with further essential enhancement of the Ω_2 frequency. Note once again that this detrimental effect is in compliance with the unexpected field observations cited in paper [3] and consisting in essential increase in the whirl frequency which can be 30 times greater than the angular velocity of the DS, while the generally recognized model of the bit touch with the bore-hole wall [2] cannot reveal it.

The established correspondences between the values of parameter p and types of the spherical bit whirling allow one to simulate the modes of its motion at the initial stage of its perturbation. But now, the further interest is provoked by the question of how the p values change with varying of the system parameters. Among the most essential values affecting the whirling stimulation are bending stiffness EI of the DS, radii of the bit a and well bottom surface b, spans l and e, WOB force T, and correlations between parameters of elasticity w_M, w_Q, $w_M^{(1)}$ and $w_Q^{(1)}$. Assume that typical magnitudes of these values for steel DSs comprise $EI = 4.07 \cdot 10^6$ Pa·m^4; $l = 9$ m; $e = 2$ m; $a = 0,1$ m; $b = 0,3$ m; $w_M = 1,968 \cdot 10^{-6}$ m; $w_Q = 3,618 \cdot 10^{-6}$ m; $w_M^{(1)} = 1,23 \cdot 10^{-6}$; $w_Q^{(1)} = 1,97 \cdot 10^{-6}$; $T = -1 \cdot 10^4 N$. In this case, $p = 1,376$ and the whirling proceeds with the frequencies $\Omega_1 = 1,376 \omega$ in the rotating reference frame and $\Omega_2 = 0,376 \omega$ in the fixed coordinate system.

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