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SOME STATIONARY DEFORMATION PROBLEMS FOR COMPOUND SHELLS OF REVOLUTION

A common approach to solving stationary deformation problems for compound systems composed of shells of revolution with different geometry and structure is developed. The approach is based on the use of shell models with different level of rigor and of the general numerical-analytical technique for solving corresponding problems. The examples of studying the subcritical stress-strain state, vibrations, and dynamical instability of complex form systems are presented, features of their deformation are noted.

Keywords: Compound systems, stationary deformation, common approach, analysis.

Introduction. Many constructions of modern engineering are modeled by the elastic systems consisting of conjugated shells of revolution with different geometry and structure. For example, they include airframes of rockets and rocket engines, underwater vehicles, protective coatings for nuclear reactors, surface tanks on cylindrical or conical supports, a.o. Study of stationary deformation of these systems covers the wide range of theoretical and applied problems. Their solving is related to reasonable compromises between the choice of the model, which describes adequately behavior of the construction on the one hand, and its effective realization for the following rational studying on the other hand. In large measure, these requirements are taken into account by the authors in developing approaches to the design of complex shell systems [1– 4].

1. Principal Ideas. The present report addresses the following three classes of problems for stationary deformation: subcritical state of flexible shells, vibrations of branched shell systems, dynamical stability of the shells undergoing harmonic actions.

These problems are united by the common object of studying and are solved using the common models of deformation and ideology of a numerical-analytical technique.

As object for study, the system consisting of J shells of revolution with different geometry, which are coaxial with respect to the 0z-axis, and including, in the general case, a number of branches from the meridian-generatrix is chosen (Fig. 1, where $\alpha = \{\alpha_j \in (\alpha_j^0, \alpha_j^{jM_j})\}$ ($j = \overline{1,J}$) is the variable that changes along the meridian).

The shells may be whether single-layer with continuous inhomogeneity of elastic properties over the thickness or consisting of several layers whose interfaces are in ideal contact (discrete inhomogeneous shells). This system undergoes actions of a stationary axisymmetric force and temperature loads, in particular, distributed along the meridian actions, as well as of forces-moments, which are concentrated on the shell ends or in a number of normal cross sections z = const. It is assumed that the layers are made of isotropic or orthotropic materials and are deformed elastically within the range of actions being considered. The ends of the shell system may be under arbitrary physically consistent boundary conditions, while the equilibrium conditions of static characteristics as well as the continuity condition for the kinematic characteris-

tics of the stress state are formulated at the lines connecting two or several shells in the general coordinate system ($r0z$).
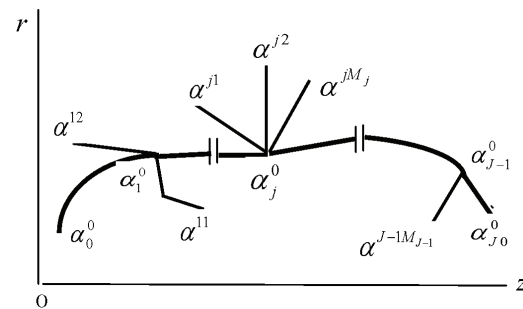

Figure 1 – General view of the generatrix- meridian of the shell system

The geometrically nonlinear mean bending theory formulated within the framework of the Kirchhoff-Love classic model, first-order shear Timoshenko-Mindlin model, and nonclassical one taking into account transverse shears and reduction is used as theoretical foundation for the analysis. The respective mathematical models are presented in the form of nonlinear one-dimensional problems, linearized two-dimensional boundary-value problems, and eigen-value different order problems.

The ideology of the numerical-analytical technique for solving corresponding problems is based on the rational reduction to one-dimensional linear boundary-value problems, which are solved numerically with high accuracy by the orthogonal-sweep method. To do this, the following methods are used: Newton–Kantorovich–Raphson quasilinearization method, reduction of dimensionality using the Fourier series, and step-by-step method as the variant of reverse iteration with formation of the Rayleigh relation.

2. Stress State of Flexible Shells. The stress-strain state of the systems composed of inhomogeneous shells of revolution within the wide range of static axisymmetric loads is considered in the subcritical stage of deformation. The study is carried out within the framework of the above prerequisites using classic and shear Timoshenko-type models and involving the quasilinearization method in combination with the orthogonal-sweep method.

Let us study the stress state of the system which consists of the following three shells of revolution (Fig. 2 *a*): cylindrical one with radius r_c, length l_c (marking C), tore-

elliptical one of negative curvature with the ellipse semi-axes a, b and distance r_{01} of the ellipse center to the axis of revolution $0z$ (marking E^-), tore-elliptical one of positive curvature with the same semi-axes and distance r_{02} of the ellipse center to the axis of revolution (marking E^+). All the shells are of the same thickness h and made of the isotropic mylare material (E is the elastic modulus, μ is Poisson's ratio) with both end contours being rigidly fixed. The $C - E^- - E^+$ system is acted upon by the external force field of the intensity q.

In problem solving, the following input data are used:

$$r_c = 0.1m, \ l_c/r_c = 1.0, \ b/r_c = 2.0, \ a/b = 0.5 - 2.0,$$
$$r_{01}/r_c = 3.0, \ r_{02}/r_c = 1.0, \ h/r_c = 0.01, \ E = 5.0285 \, \text{GPa},$$
$$\mu = 0.33; \ \alpha \in [0, \pi/2]$$

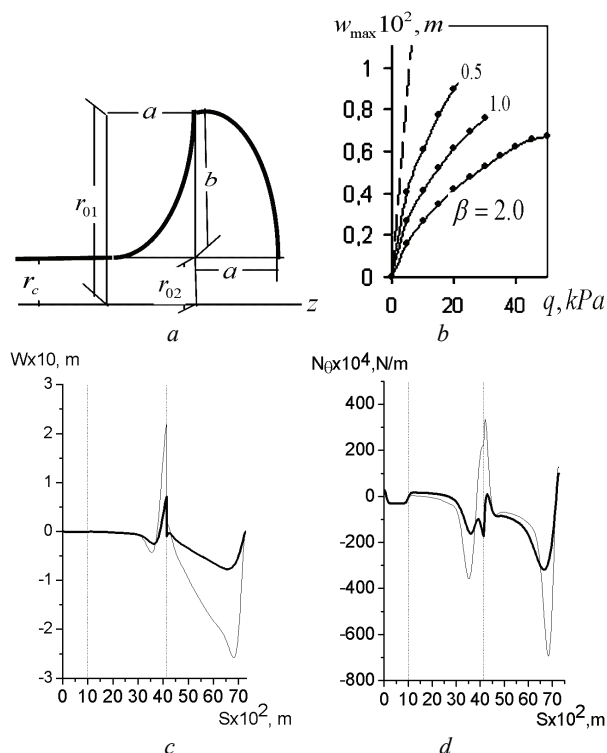(α is the central angle of both elliptical shells).

Figure 2. View of the generatrix of the system $C - E^- - E^+$ (C is cylinder, E^- is ellipse of negative curvature, E^+ is ellipse of positive curvature) (*a*); «load – deflection» curves $w_{\max}(q)$ for the ellipticity parameters $\beta = a/b = 1.0$ (*b*); distributions of the deflection $w = w(s)$ (*c*) and circumference force $N_\theta = N_\theta(s)$ (*d*) along the system generatrix s

The results obtained are presented in Fig. 2, *b* as the «load – deflection» curves $w_{\max}(q)$ for different values of the geometrical parameter $\beta = a/b = 0.5; 1.0; 2.0$ (*b* is fixed) that characterizes the ellipticity of the shells E^- and E^+. Here the strokes correspond to the linear theory for $\beta = 0.5$. The value of the limiting critical load decreases monotonically within $q_{cr} \in [25.4 - 49.8]$ kPa with β.

The distributions of the deflection w and circumferential force N_θ along the meridian-generatrix s for $\beta = 1.0$ near the critical load $q/q_{cr} = 0.93$ are shown in Fig. 2, *c*, *d*, respectively (heavy lines are referred to the nonlinear

theory, thin lines to the linear theory, normal sections $s = $ const of the conjugated elements of the system are marked by vertical straight lines). The qualitative character of these curves both defined by the linear and nonlinear theories is same for deflections as well as for stresses while quantitatively the maximum values of these functions differ more than two times.

Figure 2, *d* shows that the compressive stresses at the given external load peak in the zone of the positive curvature of the elliptical shell E^+. For this reason, this zone is the most dangerous link considering the stability of the whole system. Let us assess the possibility to define the limiting value of the load for the $C - E^- - E^+$ system as a whole by this single the most dangerous element. Values of the critical load for the shell E^+ under condition of hinged supporting in the section of conjugation with the shell E^- for $\beta \in [0.5 - 2.0]$ are collected in Table 1. Values of the limiting load for the $C - E^- - E^+$ system are presented here also.

Table 1. Values of the critical load for the tore-elliptical element with positive curvature (E^+) and for the system as a whole ($C - E^- - E^+$)

	q_{cr}, kPa			
$\beta = a/b$	0.5	1.0	1.5	2.0
E^+	9.43	15.2	21.6	28.4
$C - E^- - E^+$	25.4	32.2	41.9	49.8

As is seen from the Table, values of the limiting load for the system as a whole exceed appropriate values for the above shell by 1.5-2.5 times. This example demonstrates that all the elements of the shell system are interrelated in determining limiting critical loads.

3. Vibrations of the Branched Shells of Revolution. To study vibrations of the compound systems composed of branched shells, we involve the nonclassical model which takes into account transverse shears and reduction and employs the step-by-step method with reverse iteration in combination with the orthogonal-sweep method.

Consider the fragment of the mine pipeline with like periodically arranged external circular ribs spaced by l. The design schema of the fragment is presented by the shell system, which consists of the cylindrical shell of length l, midsurface radius R, thickness h and of the circular plate with the inner radius R, outer radius $R + l$ and thickness h conjugated with this shell in the section $z = l/2$ (Fig. 3 *a*).

The symmetry conditions are specified at the cylinder ends $z = 0$ and $z = l$. It is assumed that the outside contour of the plate $x = R + l$ is free, the conjugation conditions are given in the section of the cylinder $z = l/2$ and at the inside contour of the plate $x = R$. The cylinder and circular plate are made of an isotropic material with elastic modulus and Poisson's ratio E_c, μ_c and E_{pl}, μ_{pl} respectively.

The lowest frequencies of the shell system were studied depending on the relative stiffness of its elements with the parameter $\eta = E_{pl}/E_c \in [-1; 5]$ with $E_c = $ fixed. When $\eta < -1$ (the first limiting case), vibrations of the

system are governed only by the plate while the cylinder may be considered as the element that provides rigid fixation of its internal contour. If $\eta \geq 4$ (the second limiting case), to the contrary, the cylinder dominates while the plate can be considered as the element that provides rigid fixation of the half-length cylinder.

The results of these studies ($R = 0,1\,\mathrm{m}$, $h_c = h_{pl} = 5 \cdot 10^{-3}$, $l_c = l_{pl} = 0,1\,\mathrm{m}$; $E_c = 2 \cdot 9.81 \cdot 10^4\,\mathrm{MPa}$, $\mu = 0.3$) for the frequency parameter $\lambda = \lambda(\eta)$ are presented in Fig. 3 b ($\eta \in [1.0 - 5.0]$, $\lambda = \omega^2$, where ω is the minimal natural frequency). As is seen from the Figure, the system vibrates as a single whole only within the interval $\eta \in (2,25; 3,75)$. Outside of this interval, the system vibrates whether as a cantilever circular plate ($\eta < 2.25$) or as a half-length cylinder with the rigidly clamped contour $s = l_c / 2$ ($\eta > 3.75$).
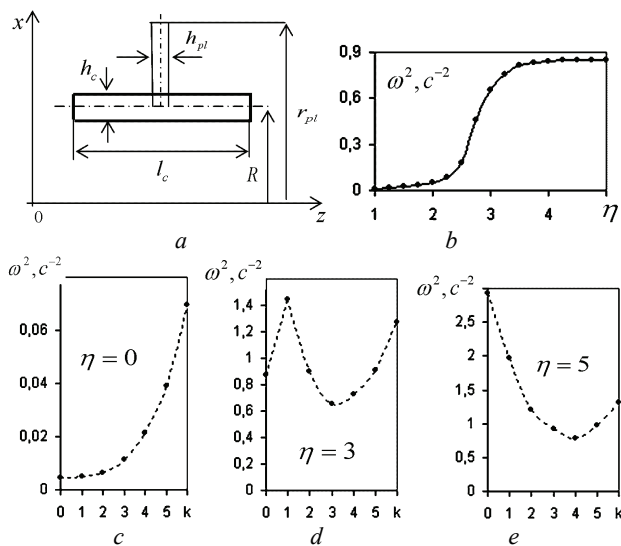

Figure 3 – Design schema of the fragment of the cylindrical pipe with circular plate (*a*); dependency of the frequency parameter $\lambda = \omega^2$ on the parameter η of the relative stiffness of the cylinder and plate (*b*); dependency $\lambda = \lambda(k)$, where k is a number of the nodal diameters of the form of the system vibrations in the circumferential direction, for $\eta = 1$ (*c*), $\eta = 3$ (*d*), $\eta = 5$ (*e*)

Note that conditional dependencies $\lambda = \lambda(k)$, which are typical for shells of revolution, demonstrate within the above intervals of variation of η qualitatively different character (see Fig. 3 *c, d, e*; k is a number of nodal diameters in the circumferential direction). The dependency $\lambda = \lambda(k)$ at $\eta = 1$ (Fig. 3 *c*) is typical for bending vibrations of plates. It is monotonic function which increases with a number of the diameters k. This dependency at $\eta = 5$ (Fig. 3, *e*) is typical for smooth cylindrical shells and is nonmonotone being minimal at $k = 4$. The dependency $\lambda = \lambda(k)$ at $\eta = 3$ (Fig. 3, *d*) takes an intermediate position between the two above considered cases and consists of two branches. The first branch at $k = 0$, $k = 1$ is a monotonically increasing functions that corresponds to prevailing vibrations of the plate $A_{pl} > A_c$ in the whole shell system (A_{pl} and A_c are the relative maximal vibration amplitudes of the plate and cylinder, respectively). The second branch at $k > 2$ is presented by the nonmonotone function

with the minimum at $k = 3$ which indicates that vibrations of the cylinder are prevailing ($A_c > A_{pl}$).

Thus, we can conclude that in the case of branched shell systems the known dependencies typical for vibrations of their single components can undergo not only quantitative changes but and qualitative ones.

4. Dynamical Stability of Shells of Revolution. Let us consider the main domain of the dynamical instability (DDI) of shells of revolution undergoing the axisymmetric periodical actions $P(s,t) = P_0(s) + P_t \cos \omega t$, where s is the length of the generatrix-meridian, $P_0(s)$ is the static component of the action, ω and $P_y(s)$ are the frequency and amplitude of its dynamical portion, t is the time variable. In studying, the following assumptions are used:

– the general state of the shell $Y = Y^0 + \delta Y$ ($Y^0 = Y_{00}(s) + Y_{0t}(s) \cos \omega t$ is the initial axisymmetric stationary state, δY is the nonaxisymmetric perturbation) is described, as above, by the two-dimensional inhomogeneous dynamical problem of the mean-bending theory without allowing for energy dissipation;

– perturbation $\delta Y(s,\theta,t)$ is defined from the homogeneous dynamical problem, which is obtained by linearization of the initial problem and contains the components of the initial axisymmetric state Y^0 with allowance for its inhomogeneity in the form of parametric terms;

– the existence conditions of periodic solutions of the homogeneous linearized problem with periods $T = 2\pi / \omega$ and $2T$; these periods correspond to the boundaries of the main DDI in approximating them by trigonometric series over the circumferential coordinate and time for each number of the harmonic k and are determined from solving two one-dimensional two-parametric eigenvalue problems.

In what follows, we will study features of the main DDI of shells with alternating Gaussian curvature in comparison with the shells of constant (zero) curvature using as an example corrugated and smooth cylinders.

Assume that the generatrix-meridian of a corrugated shell is described by a plane sinusoid curve with an amplitude a and period l whose coordinate axis is distant by R from the z-axis of revolution. At $a = 0$, we deal with the cylindrical shell of the radius R. The both shells are isotropic, of same thickness h and length $2L$ along the z-axis, ends are hinged, and in the central section $z = 0$ are subject to the shearing harmonic force with the amplitude $P_t = Q^*$ and frequency ω.

Figure 4 shows distributions of the meridional c and circumferential $\tilde{N}_\theta = N_\theta E_0 / Q^*$ forces along the generatrix for the smooth (Fig. 4 *a, I*) and corrugated (Fig. 4 *b, II*) cylinders for $h / R = 0.05$; $2L / R = 2$; $a / R = 0.16$; $2L / l = 4$; $R = 500 l_0$; $E = E_0$; $\mu = 0.3$; $\rho = \rho_0$, where l_0 is the characteristic linear dimension, E is the elastic modulus, μ is Poisson's ratio, ρ is the material density. As is seen, the meridional forces in the cylindrical shell are absent while the circumferential forces are tensile and concentrated in the vicinity of the place where the load is applied. In the case of corrugated shell, we can observe alternation of tensile and compressive zones both for me-

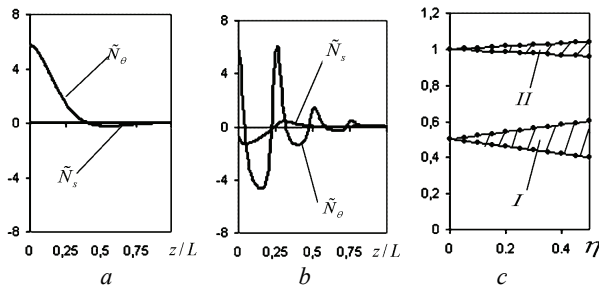ridional and circumferential forces with the circumferential ones exceeding considerably the meridional.

Figure 4 – Distributions of the meridional $\tilde{N}_s$ and circumferential $\tilde{N}_\theta$ forces for the cylindrical (*a*) and corrugated (*b*) shells and their main DDIs (*c*)

The main DDIs for these two shells are presented in Fig. 3 *c* with axes $\lambda = \omega(\eta)/\omega_{II}(0)$ and $\eta = P_t/P_{cr} \in [0;0.5]$, where $\omega_{II}(0)$ is the natural frequency of the shell with alternating curvature. As comparison of the domains presented above shows, the DDI for the corrugated cylinder (*II*) is arranged higher on the frequency axis and is significantly narrower than for the smooth cylinder (*I*).

Thus, the stress state of the shells of alternating curvature is defined by the existence of the zones with tensile and compressive stresses that, as distinct from the shells with constant curvature, may cause the loss of stability both under internal and external pressures. Besides, employing of such shells makes it possible to decrease sufficiently the domain of dangerous parameters of the harmonic action.

Conclusions. The common approach to solving a number of problems on stationary deformation of the compound systems composed of shells of revolution is proposed. The approach involves shell models with different degree of severity and general numerical-analytical technique to solve the relevant problems by the rational reduction of them to one-dimensional linear boundary-value problems.

Using the illustrative examples, the interrelation of all elements of the shell system in determining limiting critical loads as well as the possibility of qualitative change of the known dependencies typical for vibrations of its single components are shown.

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