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A NEW NUMERICAL METHOD FOR DETERMINATION OF EFFECTIVE ELASTIC CONSTANTS IN A COMPOSITE WITH CROSS-PLY FIBERS

In this paper a composite material with similar cross-ply fibers is considered. Assuming orthotropic structure, theory of elasticity is used for investigating the stress concentration. The effective characteristics of this composite are studied numerically by using ANSYS software. In this research a volume element of fibers in square array in the coordinate x, y, z and the generalized stress state is considered. In order to investigate the numerical finite element modeling, the modeling of a quarter unit cell is considered. For determining the elasticity coefficients, stress analysis is performed for considered volume with noting to boundary conditions. Effective elasticity and mechanical properties of composite which polymer epoxy is considered as its matrix, are determined theoretically and also by the proposed method in this paper with finite element method. Numerical experiments modeled four cases of uniaxial tension in the directions x, z and shear in the planes xy, yz. Finally, the variations of mechanical properties with respect to fiber-volume fraction are studied. Numerical results are compared with approximate estimates method proposed.

Keywords: composite, cross-ply fibers, effective elastic constants, orthotropic.

1 Introduction

Composite materials which consist of two or more constituent materials are commonly used in advanced structural applications, e.g. in the marine and aerospace industry. This is because of appropriate mechanical properties such as high specific strength and stiffness, low density and high resistance to corrosion. However, the limited understanding of the composite material behavior requires more research. This is further complicated by the fact that these materials behavior is dependent on lay-up, loading direction, specimen size and environmental effects such as temperature and moisture.

Research on determination of effective elastic constants for anisotropic materials is very important in composite structures.

Cross-ply laminate fiber reinforced resin matrix composites are used in some structural applications, due to their various reasons especially to their excellent mechanical behavior in terms of their specific stiffness in the direction of the fibers. The prediction of the mechanical properties of cross-ply composites has been the main objective of many researchers. The well-known models that have been proposed and used to evaluate the properties of cross-ply laminate composites are Voigt, 1989 and Reuss, 1829 models. The Voigt model is also known as the rule of mixture model or the iso-strain model, while the Reuss model is also known as the inverse of mixture model or the iso-strain model. Semi empirical models have emerged to correct the rule of mixture model where correcting factors are introduced. Under this category, it is noticed three important models: the modified rule of mixture, the Halpin-Tsai model (Halpin et al, 1976) and Chamis model (Chamis, 1989). The Halpin-Tsai model emerged as a semi-empirical model that tends to correct the transverse Young's modulus and longitudinal shear modulus. The Chamis micromechanical model is the most used and trusted model which give a formulation for all five independent elastic properties. Hashin and Rosen (Hashin et al., 1964) initially proposed a composite cylinder assemblage model to evaluate the elastic properties of cross-ply laminate composites. Christensen, 1990 proposed a generalized self-consistent model in order to better evaluate the transversal shear modulus. Also the Mori-Tanaka model (Mori et al., 1990) is a famous model which is widely used for modeling different kinds of composite materials. This is an inclusion model where fibers are simulated by inclusions embedded in a homogeneous medium. The self-consistent model has been proposed by Hill, 1965 and Budiansky, 1965 to predict the elastic properties of composite materials reinforced by isotropic spherical particulates. Later the model was presented and used to predict the elastic properties of short fibers composites (Chou et al., 1980). Recently, a new micromechanical model has been proposed by Huang, 2001. The model is developed to predict the stiffness and the strength of cross-ply laminate composites.

In this paper a composite with cross-ply fibers is considered. Assuming orthotropic structure and using ANSYS software, effective characteristics of this composite are studied. Numerical studies are performed for some stress states in a representative cell for determination the effective elastic properties of cross-ply laminate reinforced composite.

2 Computational procedure

2.1 Definition and elasticity effective parameters in orthotropic composite

This study considers a composite material with cross-ply fibers, as shown in Fig. 1. As it is shown, cross-ply fibers are parallel to «x» and «y» directions.

Theory of elasticity can be used for investigating the stress concentration of composite materials with cross-ply laminate fibers. The generalized Hook's law relating strains to stresses can be written as follows:

$$\left\langle \varepsilon_{ij} \right\rangle = \left[a_{ijkl} \right] \left\langle \sigma_{kl} \right\rangle, \quad (\mathrm{i,j} = 1,2,3), \tag{1}$$

where $[a_{\mathrm{ijkl}}]$ is the compliance matrix and $\left\langle \varepsilon_{ij} \right\rangle$ and $\left\langle \sigma_{ij} \right\rangle$ are the strain and stress components, respectively. Proof of the form of the stress-strain relations for the various cases of material property symmetry is given by Hill, 1965. For example, if there are two orthogonal planes of material property symmetry for a material, the stress-strain relations in coordinates aligned with principal material directions are as follows and are said to define an orthotropic material.

In this study, composites with cross-ply fibers and constant radius are investigated as orthotropic materials.

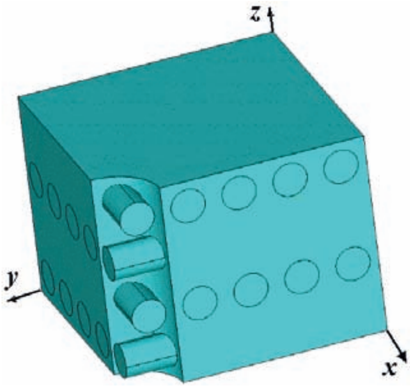


Figure 1 – Schematic of composite with cross-ply fibers

These materials with volume «*V*», stress and strain are described as follows:

$$\left\langle \sigma_{ij} \right\rangle = \frac{1}{V} \int_V \sigma_{ij} dV \qquad \text{and} \qquad \left\langle \varepsilon_{ij} \right\rangle = \frac{1}{V} \int_V \varepsilon_{ij} dV \, . \tag{2}$$

In Cartesian coordinates, Hook's law is as follows:

$$\left\langle \sigma_x \right\rangle = b_{11} \left\langle \varepsilon_x \right\rangle + b_{12} \left\langle \varepsilon_y \right\rangle + b_{13} \left\langle \varepsilon_z \right\rangle ;$$

$$\left\langle \sigma_y \right\rangle = b_{21} \left\langle \varepsilon_x \right\rangle + b_{22} \left\langle \varepsilon_y \right\rangle + b_{23} \left\langle \varepsilon_z \right\rangle ;$$

$$\left\langle \sigma_z \right\rangle = b_{31} \left\langle \varepsilon_x \right\rangle + b_{32} \left\langle \varepsilon_y \right\rangle + b_{33} \left\langle \varepsilon_z \right\rangle ;$$

$$\left\langle \tau_{xy} \right\rangle = b_{44} \left\langle \gamma_{xy} \right\rangle ; \; \left\langle \tau_{yz} \right\rangle = b_{55} \left\langle \gamma_{yz} \right\rangle ; \; \left\langle \tau_{zx} \right\rangle = b_{66} \left\langle \gamma_{zx} \right\rangle . \tag{3}$$

Where b_{ij} s the coefficient of stiffness matrix composite material. The stiffness matrix is symmetric so, $b_{ij} = b_{ji}$.

Since the composite has the same elasticity properties in «*y*» and «*z*» directions:

$$b_{11} = b_{22}; \quad b_{13} = b_{23}; \quad b_{55} = b_{66}. \tag{4}$$

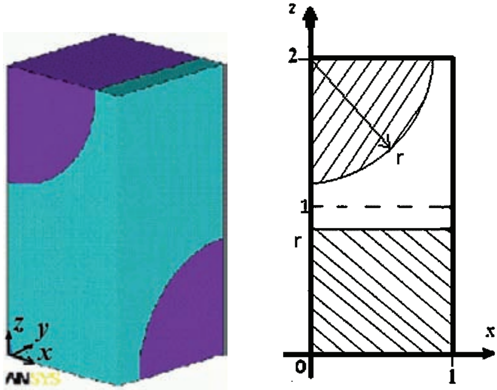
2.2 Finite Element Modeling

The numerical finite element modeling is widely used in predicting the mechanical properties of composites. In this paper for numerical analysis, a volume element of fibers in square array is considered which plane symmetric exists on all of its planes. In order to investigate the numerical finite element modeling, the modeling of a unit cell for a square array is considered using ANSYS software as shown in Fig. 2.

For determining the components of the stiffness matrix (b_{ij}), stress analysis is performed for considered volume with noting to boundary conditions. In the present procedure, normal strains are applied to two directions and shear strains are applied to two planes as follows.

The first numerical testing is unidirectional tension in «x» direction. In this condition, tensor of average values for strains is as follows:

$$\langle \varepsilon_x \rangle = 10^{-3}\,; \quad \langle \varepsilon_y \rangle = 0\,; \quad \langle \varepsilon_z \rangle = 0\,; \quad \langle \gamma_{xy} \rangle = 0\,; \quad \langle \gamma_{yz} \rangle = 0\,; \quad \langle \gamma_{xz} \rangle = 0\,. \qquad (5)$$



a *b*

Figure 2 – Model: *a* – volume element; *b* – «*xy*» plane

Boundary conditions for this structural analysis are as follows:

In plane $x = 1$: $\tau_{xy} = \tau_{xz}$ and $u_x = 10^{-3}$.

Where u_x is displacement in «*x*» direction.

In this situation, there are symmetric conditions in other planes and the boundary conditions are as follows:

$$u_x(x=0,y,z)=0\,; \quad u_y(x,y=0,z)=0\,; \quad u_y(x,y=1,z)=0\,;$$

$$u_z(x,y,z=0)=0\,; \quad u_z(x,y,z=2)=0\,,$$

where u_y, u_z are displacements in «*y*» and «*z*» directions, respectively.

For numerical analysis, finite element software ANSYS is used and 15439 SOLID 95 elements with 20 nodes are utilized as shown in Fig. 3.

Stress components are determined as:

$$\langle \sigma_x \rangle = \frac{1}{2}\int_0^2\int_0^1 \sigma_x dydz\,; \quad \langle \sigma_y \rangle = \frac{1}{2}\int_0^2\int_0^1 \sigma_y dxdz\,; \quad \langle \sigma_z \rangle = \int_0^1\int_0^1 \sigma_z dxdy\,. \qquad (6)$$

Therefore, according to equations 8, by the first numerical testing three coefficients of elasticity can be determined as follows:

$$b_{11} = \frac{\langle \sigma_x \rangle}{\langle \varepsilon_x \rangle}\,; \quad b_{21} = \frac{\langle \sigma_y \rangle}{\langle \varepsilon_x \rangle}\,; \quad b_{31} = \frac{\langle \sigma_z \rangle}{\langle \varepsilon_x \rangle}\,. \qquad (7)$$

Numerical analysis of volume element causes to study about stress-strain and stress concentration. In this analysis polymer epoxy is considered as matrix and its mechanical properties are as follows:

$E_m = 4200$ MPa; $G_m = 1500$ MPa; $\nu_m = 0{,}4$.

Mechanical properties of fibers are as follows:

E_a = 74800 MPa; G_a = 31000 MPa; ν_a = 0,2 [3,6].

For composite with fibers in constant radius as $0 < r < 1$, fiber-volume fraction is calculated as follows for square array:

$$\xi = \frac{1}{4}\pi r^2 . \tag{8}$$

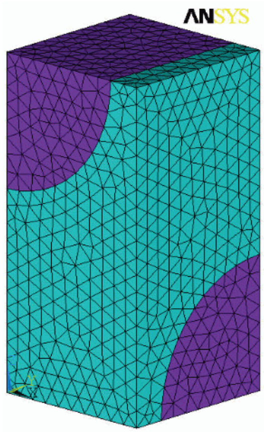


Figure 3 – Volume element in mesh formed

Effective elasticity properties for $\xi = 0{,}488$ is determined by Numerical Method. In Fig. 4 the result of the first condition is shown.

The second numerical testing is unidirectional tension in «z» direction. In this condition, tensor of average values for strains is as follows:

$$\langle \varepsilon_x \rangle = 0\,; \quad \langle \varepsilon_y \rangle = 0\,; \quad \langle \varepsilon_z \rangle = \frac{10^{-3}}{2}\,; \quad \langle \gamma_{xy} \rangle = 0\,; \quad \langle \gamma_{yz} \rangle = 0\,; \quad \langle \gamma_{xz} \rangle = 0\,. \tag{9}$$

Boundary conditions for this structural analysis are as follows:

In plane $z = 2$: $u_z = 10^{-3}$; $\tau_{xz} = \tau_{zy}$.

In this situation, there are symmetric conditions in other planes and the boundary conditions are as follows:

$$u_z(x, y, z = 0) = 0\,; \quad u_x(x = 0, y, z) = 0\,; \quad u_x(x = 1, y, z) = 0\,;$$

$$u_y(x, y = 0, z) = 0\,; \quad u_y(x, y = 1, z) = 0\,.$$

In this situation, the following equation is also obtained:

$$\langle \sigma_z \rangle = \int_0^1 \int_0^1 \sigma_z\, dxdy\,. \tag{10}$$

Therefore, according to equations 8, by the second numerical testing one of the coefficients of elasticity can be determined as follows:

$$b_{33} = \langle \sigma_z \rangle / \langle \varepsilon_3 \rangle\,. \tag{11}$$

The third numerical testing is shearing in «xy» plane. In this condition, tensor of average values for strains is as follows:

$$\langle\varepsilon_x\rangle = 0\,;\quad \langle\varepsilon_y\rangle = 0\,;\quad \langle\varepsilon_z\rangle = 0\,;\quad \langle\gamma_{xy}\rangle = 10^{-3}\,;\quad \langle\gamma_{yz}\rangle = 0\,;\quad \langle\gamma_{xz}\rangle = 0\,. \qquad (12)$$

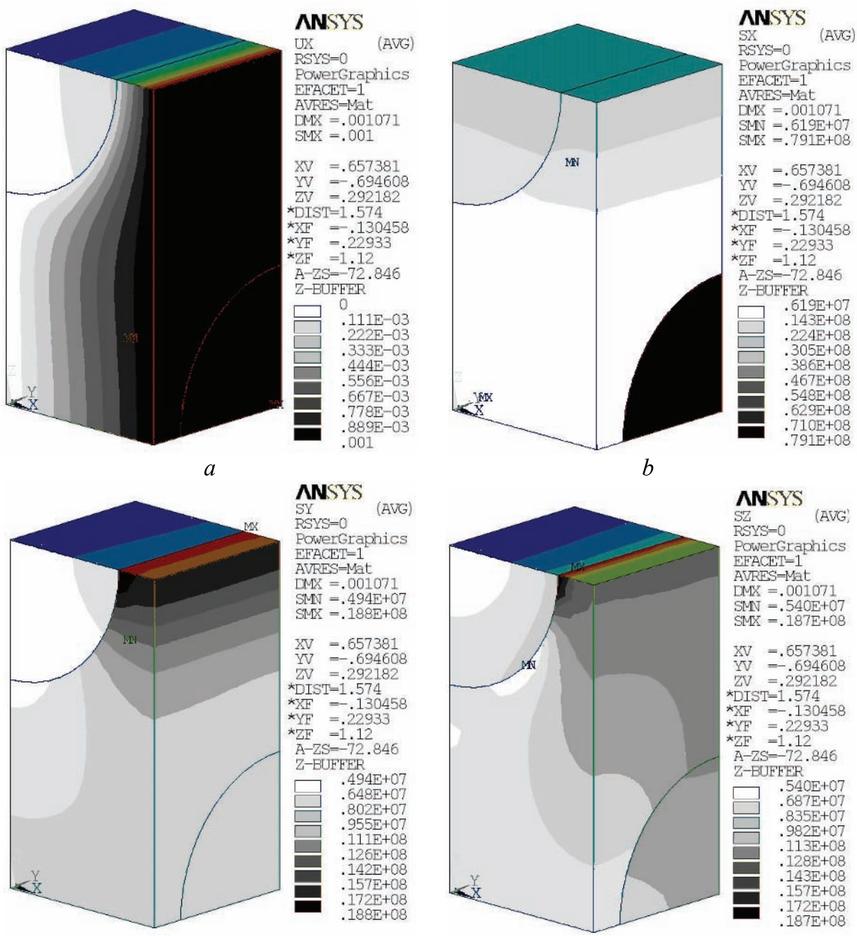


Figure 4 – Results: *a* – displacement in «x» direction when $u_x = 10^{-3}$;
b, *c* and *d* – stress distribution of normal stresses σ_x, σ_y, σ_z

Boundary conditions for this structural analysis are as follows:

In plane $x = 1$: $u_y = 10^{-3}$; $\sigma_x = \tau_{xz} = 0$ and $u_y = 10^{-3}$.

Where u_y is displacement of plane «$x = 1$» in «y» direction.

In plane «$x = 1$» the displacement in all of the directions are zero and there are symmetric conditions in planes «$x = 1$», «$z = 0$» and «$z = 2$». The shear stress component in «xy» plane is as follows:

$$\left\langle \tau_{xy} \right\rangle = \frac{1}{2}\int_0^2\int_0^1 \tau_{xy}\,dydz\ . \qquad (13)$$

Therefore, according to equations 8, by the third numerical testing another coefficient of elasticity can be determined as follows:

$$b_{44} = \left\langle \tau_{xy} \right\rangle / \left\langle \gamma_{xy} \right\rangle . \qquad (14)$$

The fourth numerical testing is shearing in «*yz*» plane. In this condition, tensor of average values for strains is as follows:

$$\left\langle \varepsilon_x \right\rangle = 0\,;\quad \left\langle \varepsilon_y \right\rangle = 0\,;\quad \left\langle \varepsilon_z \right\rangle = 0\,;\quad \left\langle \gamma_{xy} \right\rangle = 0\,;\quad \left\langle \gamma_{yz} \right\rangle = 10^{-3}/2\,;\quad \left\langle \gamma_{xz} \right\rangle = 0\,. \qquad (15)$$

Boundary conditions for this structural analysis are as follows:

In plane z = 2: $\sigma_z = \tau_{zx} = 0$ and $u_y = 10^{-3}$.

Where $u_y = 10^{-3}$ is displacement of plane «z = 2» in «y» direction.

In plane «*z* = 0» the displacement in all of the directions are zero and there are symmetric conditions in planes «*x* = 0», «*x* = 1» and «*z* = 2». The stress component in «*yz*» plane is as follows:

$$\left\langle \tau_{yz} \right\rangle = \int_0^1\int_0^1 \tau_{yz}\,dxdy\ . \qquad (16)$$

Therefore, according to equations 8, by the fourth numerical testing the last coefficient of elasticity can be determined as follows:

$$b_{55} = \left\langle \tau_{zy} \right\rangle / \left\langle \gamma_{zy} \right\rangle . \qquad (17)$$

For solving the problem, Hook's law is used directly:

$$\left\langle \varepsilon \right\rangle = [A]\cdot\left\langle \sigma \right\rangle . \qquad (18)$$

where A is the compliance matrix: $[A] = [B]^{-1}$.

Considering matrix *A*, elasticity coefficient such as poison ratio and shear modulus can be obtained as follows:

$$a_{11} = \frac{1}{E_x};\quad a_{22} = \frac{1}{E_y};\quad a_{33} = \frac{1}{E_z};\quad a_{12} = -\frac{\nu_{yx}}{E_y} = -\frac{\nu_{xy}}{E_x};\quad a_{13} = -\frac{\nu_{zx}}{E_z} = -\frac{\nu_{xz}}{E_x};$$

$$a_{23} = -\frac{\nu_{zy}}{E_z} = -\frac{\nu_{yz}}{E_y};\quad G_{xy} = b_{44}\,;\quad G_{yz} = b_{55}\,;\quad G_{zx} = b_{66}\,. \qquad (19)$$

Complex variable functions are used for solving the plane stress problems [13]. In this procedure, the elasticity coefficients of composite structures are dependent to the material properties of matrix and fibers and also to the situation of the fibers in the matrix. Vanin, 1985 determined the properties of composites with unidirectional fibers by complex functions. These coefficients are as follows:

$$\left\langle E_x^0 \right\rangle = \xi\, E_b + (1-\xi)E_m + \frac{8G_m\xi(1-\xi)(\upsilon_b-\upsilon_m)}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b};$$

$$\frac{1}{\left\langle E_y^0 \right\rangle} = \frac{(\upsilon_{yx})^2}{E_x} + \frac{1}{8G_m}\left[\frac{2(1-\xi)(\chi_m-1)+(\chi_b-1)(\chi_m-1+2\xi)G_m/G_b}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b} + \right.$$

$$+2\frac{\chi_m(1-\xi)+(1+\xi\chi_m)G_m/G_b}{\chi_m+\xi+(1-\xi)G_m/G_b}\Bigg];$$

$$\left\langle G_{xy}^0\right\rangle = G_m\frac{1-\xi+(1-\xi)G_m/G_b}{1-\xi+(1+\xi)G_m/G_b}; \quad \left\langle G_{yz}^0\right\rangle = G_m\frac{(1-\xi)\chi+(1+\chi\xi)G_m/G_b}{\chi+\xi+(1-\xi)G_m/G_b};$$

$$\left\langle \upsilon_{yx}^0\right\rangle = \upsilon_m - \frac{(\chi_m+1)(\upsilon_m-\upsilon_b)\xi}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b}, \tag{20}$$

$$\left\langle \upsilon_{xy}^0\right\rangle = \left\langle \upsilon_{yx}^0\right\rangle\frac{\left\langle E_y^0\right\rangle}{\left\langle E_x^0\right\rangle}; \quad \left\langle \upsilon_{zy}^0\right\rangle = \left\langle \upsilon_{yz}^0\right\rangle\frac{\left\langle E_z^0\right\rangle}{\left\langle E_y^0\right\rangle}; \quad \left\langle \upsilon_{zx}^0\right\rangle = \left\langle \upsilon_{xz}^0\right\rangle\frac{\left\langle E_z^0\right\rangle}{\left\langle E_x^0\right\rangle}. \tag{21}$$

where $\chi_i = 3-4\upsilon_i$, ($i = m,a$) and ξ – fiber-volume fraction for a unidirectional fibers composite.

In this study fibers are parallel and perpendicular to «x» direction. So, for an orthotropic material:

$$\left\langle E_z^0\right\rangle = \left\langle E_y^0\right\rangle; \quad \left\langle G_{xy}^0\right\rangle = \left\langle G_{zx}^0\right\rangle; \quad \left\langle \upsilon_{zx}^0\right\rangle = \left\langle \upsilon_{yx}^0\right\rangle; \quad \left\langle \upsilon_{yz}^0\right\rangle = \left\langle \upsilon_{zy}^0\right\rangle; \quad \left\langle \upsilon_{xz}^0\right\rangle = \left\langle \upsilon_{xy}^0\right\rangle.$$

In the above equations, $\left\langle E_x^0\right\rangle$, $\left\langle E_y^0\right\rangle$, $\left\langle E_z^0\right\rangle$ – Young's moduli in the x-, y- and z-directions, $\left\langle G_{ij}^0\right\rangle$ – Shear modulus in the i-j plane and $\left\langle \upsilon_{ij}^0\right\rangle$ – Poisson's ratio in the i-j plane ($i,j = x,y,z$) are the mean composite material modulus for unidirectional fibers material and E_m, G_m, v_m and E_b, G_b, v_b are the matrix and fiber's coefficients, respectively.

According to Vanin's equations, Alfootov determined the mechanical properties of cross-ply laminate reinforced composites with perpendicular fibers by complex functions. Considering a two layer cross-ply laminate of equal properties with fibers parallel to «x» and «y» directions he obtained the mechanical properties of one layer and then the results were extended to multilayered plates. In this research multilayered plates are assumed with unit length and width.

The stress-strain relations for a plate with similar unidirectional fibers under plane stress are as follows:

$$\left\{\left\langle \sigma_{ij}\right\rangle\right\} = \left[B^0\right]\left\{\left\langle \varepsilon_{ij}\right\rangle\right\}, \quad (i,j = 1,2).$$

where, $[B^0]$ is the stiffness matrix of plate with similar unidirectional fibers. The above equation can be written as:

$$\begin{Bmatrix}\left\langle\sigma_{11}\right\rangle\\ \left\langle\sigma_{22}\right\rangle\\ \left\langle\sigma_{12}\right\rangle\end{Bmatrix} = \begin{bmatrix} b_{11}^0 & b_{12}^0 & 0\\ b_{21}^0 & b_{22}^0 & 0\\ 0 & 0 & b_{44}^0\end{bmatrix}\begin{Bmatrix}\left\langle\varepsilon_{11}\right\rangle\\ \left\langle\varepsilon_{22}\right\rangle\\ \left\langle\varepsilon_{12}\right\rangle\end{Bmatrix} \tag{22}$$

in laminates with multiple orthogonal layers and perpendicular fibers, medium thickness plates is considered. In this case the directions of the principal axes are assumed to coincide to «x» and «y» axes and all the layers are made of the same stiffness characters.

Stiffness matrix of this material describes the orthotropic properties of multi-layered plates with orthogonal fibers. So, the coefficients of elasticity are as follows:

$$b_{11} = \frac{1}{2}\left(b_{11}^{0} + b_{22}^{0}\right); \quad b_{22} = \frac{1}{2}\left(b_{22}^{0} + b_{11}^{0}\right); \quad b_{12} = b_{12}^{0}; \quad b_{44} = b_{44}^{0}. \tag{23}$$

Effective elasticity properties for $\xi = 0{,}488$ is determined by theory of complex functions and numerical procedure proposed in this research. Table 1 shows theoretical and numerical effective elastic constants. Numerical values are calculated by ANSYS.

3 Results and discussion

In this section, variation of $E_1 = \langle E_x \rangle / E_m$, $E_2 = \langle E_y \rangle / E_m$ and $G = \langle G_{xy} \rangle / G_m$ versus different values of ξ are obtained for cross-ply laminate glass fibers in a square pattern. Mechanical properties of composite are determined theoretically (method Vanin-Alfootov) and also by the proposed method in this paper (numerical method) with finite element method software (ANSYS). It is obvious that $\langle E_x \rangle = \langle E_y \rangle$, so $E_1 = E_2$. In addition, for the integration of theoretical and numerical methods to solve the problem, it is assumed that the number and thickness of layer 1 and 2 are equal.

Table 1 – Theory and numerical results of effective elasticity properties for $\xi = 0{,}488$

Elasticity properties		Numerical Method by ANSYS	Theoretical Method by Vanin Formula
Modulus of elasticity (MPa)	$\langle E_x \rangle$	30400	29470
	$\langle E_y \rangle$	30400	29470
	$\langle E_x \rangle$	16100	14870
Modulus of shear (MPa)	$\langle G_{xy} \rangle$	26500	5293
	$\langle G_{xz} \rangle$	3700	3278
	$\langle G_{yz} \rangle$	3700	3278
Poisson's coefficient	$\langle \upsilon_{xy} \rangle$	0,02	0,02
	$\langle \upsilon_{xz} \rangle$	0,31	0,286

Fig. 5 shows the variation of E_1 versus different values of ξ for cross-ply composite glass fibers in a square pattern. $\langle E_x \rangle$ is modulus of elasticity of composite in fibers direction and E_m is modulus of elasticity of matrix. In this figure, the curve 1 is obtained from theoretical formulation and the curve 2 is obtained by the method of this paper. As it can be seen, the behaviors of curves are nonlinear.

Fig. 6 shows that in small value of ξ, the value of E_1 is near to 1. Also for the maximum value of ξ (ξ = 0,78), the value of E_1 is near to the modulus of elasticity of fibers E_b/E_m, as it is predicted.

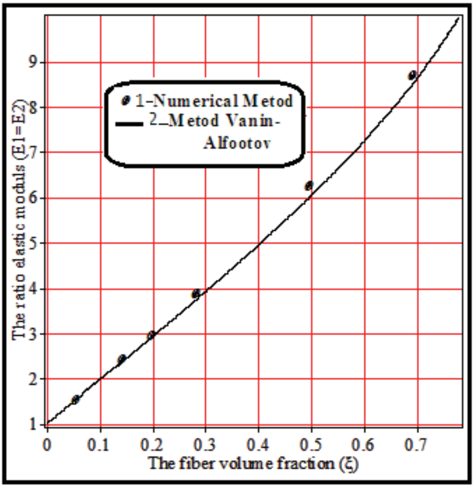


Figure 5 – The variation of E_1 and E_2 ($E_1 = E_2$) versus different values of ξ for cross-ply glass fibers in a square pattern

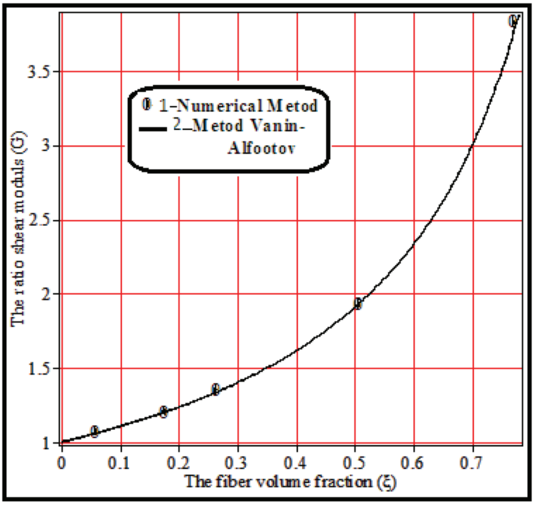


Figure 6 – The variation of G versus different values of ξ for cross-ply glass fibers in a square pattern

Fig. 6 shows the variation of G versus different values of ξ for cross-ply composite glass fibers in a square pattern. $\left\langle G_{xy} \right\rangle$ is shear modulus of composite in xy plane and G_m is the shear modulus of matrix. In this figure, the curve 1 is obtained from theoretical formulation and the curve 2 is obtained by the method of this paper. As it can be seen, the behaviors of curves are nonlinear. Fig. 6 shows that in small value of ξ, the value of G is near to 1. Also for the maximum value of ξ ($\xi = 0{,}78$), G is near to a value that is smaller than the shear modulus of fibers G_b/G_m. This result is coincident to curves obtained by Vanin- Alfootov.

Coclusion

In this research assuming orthotropic structure for a composite material with similar cross-ply fibers, the effective elasticity and mechanical properties are determined theoretically and also by finite element method. A volume element of fibers in square array is considered which plane symmetric exists on all of its planes. In order to investigate the numerical finite element modeling, the modeling of a quarter unit cell is considered. For determining the elasticity coefficients, stress analysis is performed for considered volume with noting to boundary conditions. In the present procedure, normal strains are applied to two directions and shear strains are applied to one plane. So, the effective elasticity and mechanical properties of composite which polymer epoxy is considered as its matrix, are determined theoretically and also by the proposed method in this paper.

The variations of mechanical properties with respect to fiber-volume fraction ξ are studied and the following results are obtained:

1. In direction of fibers, the behaviors of ratio $E_1 = E_2$ due to ξ are nonlinear. The results show that in small value of ξ, the value of E_1 is near to 1. Also for the maximum value of ξ, the value of E_1 and E_2 is near to the modulus of elasticity of fibers E_b/E_m, as it is predicted.

2. The behaviors of ratio G are nonlinear. The results show that in small value of ξ, the value of G is near to 1. Also for the maximum value of ξ, the value of G is near to a value that is smaller than the shear modulus of fibers G_b/G_m. This result is coincident to curves obtained by Vanin.

References: **1.** *Alfootov N.A.* Calculation of laminated plates and shells made of composite materials. – M.: Mashinostroitelni, 1984. **2.** *Budiansky B.* On the elastic modulli of some heterogeneous materials // Journal of Mechanics and Physics of solids. – 1965. – № 13. – P. 223-227. **3.** *Carpeenosa D. M.* Composite Materials. – K.: Naukova dumka, 1985. **4.** *Chamis C. C.* Mechanics of composite materials: past, present and future // J. Compos Technol Res ASTM. – 1989. – № 11. – P. 3-14. **5.** *Chou T. W., Nomura S., Taya M.* A self-consistent approach to the elastic stiffness of short-fiber composites // J. Compos Mater. – 1980. – № 14. – P. 178-188. **6.** *Christensen R. M.* A critical evaluation for a class of micromechanical models // Journal of Mechanics and Physics of Solids. – 1990. – № 38(3). – P. 379-404. **7.** *Halpin J.C., Kardos J.L.* The Halpin-Tsai equations // A review Polymer Engineering and Science. – 1976. – Vol. 16, №. 5. **8.** *Hashin Z., Rosen B. W.* The elastic moduli of fiber reinforced materials // Journal of applied Mechanics. Trans ASME. – 1964. – № 31. – P. 223-232. **9.** *Hill R.* Theory of mechanical properties of fiber-strengthen materials-3. Self-consistent model // Journal of Mechanics and Physics of solids. – 1965. – № 13. – P. 189-198. **10.** *Huang Z. M.* Micromechanical prediction of ultimate strength of transversely isotropic fibrous composites // International Journal of Solids and Structures. – 2001. – № 38. – P. 4147-4172. **11.** *Huang Z. M.* Simulation of the mechanical properties of fibrous composites by the bridging micromechanics model // Composites: Part A. – 2001. – № 32. – P. 143-172. **12.** *Mori T., Tanaka K.* Average stress in matrix and average elastic energy of materials with misfitting inclusions // Acta Mettall. – 1990. – № 21. – P. 571-574. **13.** *Reuss A*. Berechnung der Fliessgrenze von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle // Zeitschrift Angewandte Mathematik und Mechanik. – 1829. – № 9. – S. 49-58. **14.** *Vanin G. A.* Micro-Mechanics of Composite Materials. – K.: Naukova dumka, 1985. **15.** *Voigt W*. Über die Beziehung zwischen den beiden Elastizitätskonstanten Isotropen Körper // Wied Ann. – 1989. – № 38. – S. 573-587.

Bibliography (transliterated): 1. Alfootov N.A. Calculation of laminated plates and shells made of composite materials. Moscow: Mashinostroitelni, 1984. **2.** Budiansky B. On the elastic modulli of some heterogeneous materials. Journal of Mechanics and Physics of solids. 1965. 13: 223-227 Print. **3.** Carpeenosa D. M. Composite Materials. Kyyiv: Naukova dumka, 1985. **4.** Chamis C. C. Mechanics of composite materials: past, present and future. J. Compos Technol Res ASTM. 1989. 11: 3-14 Print. **5.** Chou T. W., Nomura S., Taya M. A self-consistent approach to the elastic stiffness of short-fiber composites. J. Compos Mater. 1980. 14: 178-188 Print. **6.** Christensen R. M. A critical evaluation for a class of micromechanical models. Journal of Mechanics and Physics of Solids. 1990. 38(3): 379-404 Print. **7.** Halpin J.C., Kardos J.L. The Halpin-Tsai equations. A review Polymer Engineering and Science. 1976. Vol. 16, №. 5 Print. **8.** Hashin Z., Rosen B. W. The elastic moduli of fiber reinforced materials // Journal of applied Mechanics. Trans ASME. 1964. 31: 223-232 Print. **9.** Hill R. Theory of mechanical properties of fiber-strengthen materials-3. Self-consistent model. Journal of Mechanics and Physics of solids. 1965. 13: 189-198 Print. **10.** Huang Z. M. Micromechanical prediction of ultimate strength of transversely isotropic fibrous composites. International Journal of Solids and Structures. 2001. 38: 4147-4172 Print. **11.** Huang Z. M. Simulation of the mechanical properties of fibrous composites by the bridging micromechanics model. Composites: Part A. 2001. 32: 143-172 Print. **12.** Mori T., Tanaka K. Average stress in matrix and average elastic energy of materials with misfitting inclusions. Acta Mettall. 1990. 21: 571-574 Print. **13.** Reuss A. Berechnung der Fliessgrenze von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle. Zeitschrift Angewandte Mathematik und Mechanik. 1829. 9: 49-58 Print. **14.** Vanin G. A. Micro-Mechanics of Composite Materials. Kyyiv: Naukova dumka, 1985 Print. **15.** Voigt W. Über die Beziehung zwischen den beiden Elastizitätskonstanten Isotropen Körper. Wied Ann. 1989. 38: 573-587 Print.

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