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NON-STATIONARY PHENOMENA IN TECHNOLOGICAL SYSTEMS OF ELECTROMAGNETIC PROCESSING OF MATERIALS

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A large number of technical and technological facilities work under the action of electromagnetic fields. In electroconductive bodies have significant largest electromagnetic forces that can cause movement or deformation of structural elements. The creation of effective methods of analysis of the distribution of the electromagnetic field and coupled nonstationary deformation of structural elements is topical at present time. The article contains a mathematical formulation of the problem of nonstationary deformation of structural elements under the action of electromagnetic fields. Coupling of electromagnetic field and mechanical field is carried out with the help of local electromagnetic forces. Further made the transition to a variational formulation on the basis of the task of finding the minimum of the total energy of the system, which includes the energy of the electromagnetic field. For the numerical solution the finite element method is used. Nodal unknowns in this case are the magnetic vector potential and displacements. The proposed method is applied to non-stationary deformation of the "inductor-billet" technological operation of magnetic-pulse processing of metals. Some results of the deformation are presented.

Keywords: non-stationary deformation, electromagnetic field, vector magnetic potential, electromagnetic force, the finite element method.

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НЕСТАЦІОНАРНІ ЯВИЩА В ТЕХНОЛОГІЧНИХ СИСТЕМАХ ЕЛЕКТРОМАГНІТНОЇ ОБРОБКИ МАТЕРІАЛІВ

У статті розглядаються проблеми створення ефективного методу аналізу нестаціонарного деформування елементів конструкцій під дією електромагнітного поля. Розглянуто загальну математичну постановку зв'язаної задачі деформування електропровідних тіл при наявності електромагнітного поля. Зв'язок поміж процесами розповсюдження електромагнітного поля та процесами деформування здійснюється завдяки електромагнітним силам. Для побудови методу чисельного розв'язку вихідна задача зведена до пошуку мінімуму повної енергії системи. В якості чисельного методу розв'язання використовується метод скінчених елементів. Запропонований метод застосований для аналізу нестаціонарного деформування системи „індуктор–заготівка" технологічної операції магнітно-імпульсної обробки металів. Представлені деякі результати, які дозволяють робити певні рекомендації щодо проектування та застосування технологічних операцій подібного класу.

Ключові слова: нестаціонарне деформування, електромагнітне поле, векторний магнітний потенціал, електромагнітні сили, метод скінчений елементів.

Introduction. A large number of technical and technological objects function in the presence of a time-varying electromagnetic field (EM-field). At the same time, significant electromagnetic forces arise in the structural elements made of electrically conductive materials, which lead to both the movement of the corresponding structural elements and their deformation. The problems of taking into account non-stationary processes under the action of EM-field arise in the design and operation of many objects of modern technology: generators, resonant vibro-experimental systems, technological systems for pulse magnetic processing of metals (PMPM), etc. In the latter case, both structural elements generating EM-field (so-called inductors) and workpieces being processed are under the influence of EM-field.

Analysis of recent research and publications. A review of issues related to aspects of the creation and application of devices that use the energy of EM-field for technological purposes is quite fully given in the works [1-4]. Modern directions of development of PMPM for processing non-traditional objects are presented in the works [5-7]. Taking dynamic processes into account during PMPM is especially important in the case when the workpiece is thin-walled, because in this case, under impulse loading, its irreversible shape change will necessarily be accompanied by oscillatory processes due to elastic properties [3,4].

It should be noted that in the vast majority of studies devoted to the analysis of technological operations of PMPM, the task of determining the vector components of EM-field and the task of stress-strain state (SSS) analysis are considered separately. The modern view of this problem, in our opinion, should consist in a joint analysis of EM-field propagation processes and deformation processes within the framework of a single calculation scheme, similar to how it is done in the works [8,9].

Thus, the creation of effective methods for the analysis of non-stationary elastic-plastic deformation of structural elements of complex geometry under the action of EM-field is an urgent scientific and practical problem.

The purpose of the article. The article is directly devoted to the issue of creating an effective method for the analysis of non-stationary phenomena in the elements of the technological equipment of the PMPM and workpieces.

Such phenomena include both the non-stationary behavior of the EM-field and the non-stationarity of deformation caused by the time variability of electromagnetic forces and the elastic properties of the material.

It is also necessary, using the proposed method, to conduct a study of the distribution of the spatio-temporal characteristics of the EM-field and the stress-strain state for a real technological system focused on the deformation of thin-walled structural elements.

Mathematical formulation of the problem. Any numerical method of computational analysis must be based on the correct mathematical formulation of the problem. In this case, we take as a basis the complete statement of the problem of elastic-plastic deformation of the system of conductive bodies under the action of an electromagnetic field, given in the paper [9].

Let us consider the general statement of the problem of non-stationary deformation of conductive bodies in the presence of EM-field. Electromagnetic processes of PMPM in the absence of free charges are described by the following system of Maxwell's fundamental equations:

$$rot\ \vec{H} = \varepsilon_c \frac{\partial \vec{E}}{\partial t} + \vec{j}\ ,\ rot\ \vec{E} = -\mu_c \frac{\partial \vec{H}}{\partial t}\ ,\ div\vec{H} = 0\ ,$$

$$div\vec{E} = 0\ , \qquad (1)$$

where $\vec{j},\ \vec{E},\ \vec{H}$ – current density, electric and magnetic field strengths in the subregion, μ_c, ε_c – magnetic and electrical permeability. Neglecting convection currents, equation (1) can be supplemented with material relations:

$$\vec{D} = \varepsilon_c \vec{E}\ ,\ \vec{B} = \mu_c \vec{H}\ ,\ \vec{j} = \gamma_c \vec{E} + \gamma_c \left[\dot{\vec{u}} \times \vec{B} \right], \qquad (2)$$

where $\vec{D},\ \vec{B}$ – induction vectors of electric and magnetic fields in the subregion, γ_c – specific electrical conductivity of the material.

The complete system of equations for the components of the tensors of stresses, deformations, and the vector of displacements, given the volume and surface forces, will be written in the following form:

Equations of motion:

$$\sigma_{ij,j} + F_{pi} = \rho \ddot{u}_i\ , \qquad (3)$$

where σ_{ij} are components of the stress tensor, F_{pi} – components of the vector of volume forces, which can be, among others, electromagnetic, ρ – material density, u_i – components of the displacement vector.

Geometric Cauchy relations for small deformations:

$$\varepsilon_{ij} = 1/2\left(u_{j,i} + u_{i,j}\right), \qquad (4)$$

where ε_{ij} – components of the strain tensor. Equation (1) is supplemented by boundary conditions:

$$\vec{E}_\Gamma \times \vec{n} = 0, \quad \vec{D}_\Gamma \cdot \vec{n} = 0, \quad \vec{H}_\Gamma \times \vec{n} = 0, \quad \vec{B}_\Gamma \cdot \vec{n} = 0; \qquad (5)$$

$$\vec{\sigma}_n = \vec{p}_n + \frac{\Xi}{2}\vec{E}_\Gamma + \frac{\mu_c}{2}\left(\Xi \dot{\vec{u}}_\tau + \vec{i}\right) \times \vec{H}_\Gamma\ , \qquad (6)$$

where $\vec{\sigma}_n = \sigma \cdot \vec{n}$ – vector of mechanical stresses on the boundary with the normal $\vec{n}$, $\Xi, \vec{i}$ – densities of surface charges and currents, $\dot{\vec{u}}_\tau$ – the projection of the velocity vector of a point onto the plane tangent to the boundary of the body.

Generalized equations of state establishing the relationship between stresses and deformations at the points of bodies that deform elastically can be represented by tensor-linear relations of the form:

$$\varepsilon_{ij} = A_{ijkl}\sigma_{kl} + \alpha_{ij}\Delta T\ , \qquad (7)$$

where ε_{ij}, σ_{kl} – components of strain and stress tensors, A_{ijkl} are components of the tensor used to describe the properties of the material, α_{ij} – components of the tensor of the thermal expansion properties of the material.

The given problem can be effectively solved by numerical methods, among which the finite element method (FEM) is the most universal. The principle of the minimum total energy of the system can be used as a basis for the specific use of FEM:

$$\delta E = 0, \quad E = U + W\ , \qquad (8)$$

where U – mechanical energy of deformation, W – EM-field energy. If we introduce the concept of vector magnetic potential [9]: $\vec{B} = rot\vec{A}$, then the EM-field energy and the mechanical energy can be written in vector-matrix form as follows:

$$W = \frac{1}{2}\{A\}^T [M]\{A\},$$

$$U = \frac{1}{2}\{u\}^T [K]\{u\} + \frac{1}{2}\{\dot{u}\}^T [Mass]\{\dot{u}\}, \qquad (9)$$

where $[M]$ – the "magnetic" matrix of the material, which depends on the properties of the material, on the conditions of the geometry of the body under consideration, and is determined using Maxwell's fundamental equations; $[K]$ – stiffness matrix; $[Mass]$ – mass matrix. Thus, after the variation operation in the case of neglecting magnetostriction effects, the problem of analyzing the non-stationary deformation of an electrically conductive body in the presence of an EM-field is reduced to solving a system of two equations:

$$\begin{cases} [M]\{A\} = 0; \\ [K]\{u\} = [Mass]\{\ddot{u}\} + \{F_{em}\}. \end{cases} \qquad (10)$$

where $\{F_{em}\} = -\frac{\partial W}{\partial u} = -\frac{1}{2}\{A\}^T \frac{\partial [M]}{\partial u}\{A\}$ – column vector of electromagnetic forces. Note that a similar approach to the definition of electromagnetic forces is implemented, for example, in work [3]. When applying the FEM scheme in this case, the nodal unknowns of the finite element will be the components of the vector magnetic potential and the components of the displacement vector.

Calculation example. Let's consider the application of this method to the analysis of the deformation of the "inductor-workpiece" system, the calculation scheme of which is shown in Fig. 1. In reality, the inductor is a massive body of rotation of a complex shape. The principle of operation and scope of application of such an inductor are described in detail in the paper [5,6]. It is important for us that the inductor can be considered as an axisymmetric body, and the direct source of EM-field is a current that is uniformly distributed along the line Γ_2. When solving the problem, the meridional sections of the inductor were modeled (material copper: relative magnetic permeability $\mu_r = 1$, modulus of elasticity – 180 GPa, Poisson's ratio - 0.33, yield strength of the order of 200 - 210 MPa), blanks (material steel: relative magnetic permeability $\mu_r = 1.3$, modulus of elasticity - 210 GPa, Poisson's ratio - 0.28, yield strength of the order of 250 - 280 MPa) the intermediate and surrounding medium - air (relative magnetic permeability $\mu_r = 1$). The characteristic dimensions of the interacting bodies are as follows: smaller radius of the conical opening R_1 = 10 mm; larger radius R_2 = 20 mm; overall radius inductor R_3 = 60 mm; workpiece radius R_4 = 120 mm; the thickness of the inductor is 12 mm; workpiece thickness - 1 mm; the smallest distance between the inductor and the workpiece is 1 mm. The dimensions of the surrounding air environment were chosen in such a way as to simulate the attenuation of the EM-field at a distance from its source. As is known [6], when the EM-field is attenuated, the vector magnetic potential tends to zero, so a zero magnetic potential was set at the outer boundary of Γ_3 (Fig. 1).

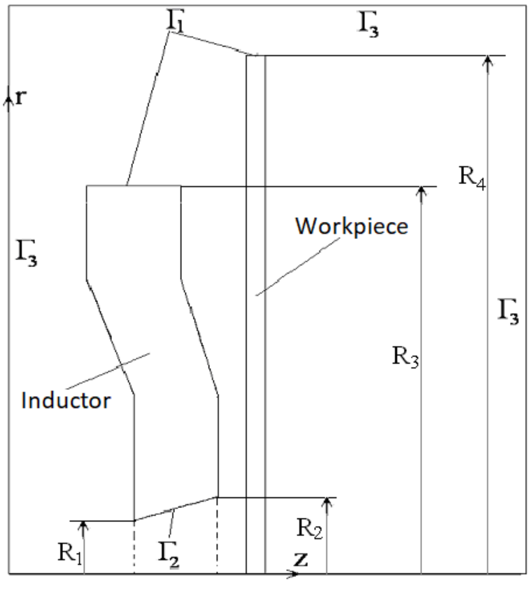

Fig. 1 – Calculation scheme

A current was set on line Γ_2, which changed over time according to the law: $I = I_m e^{-\delta_0 \omega t} \cos(\omega t)$, here $I_m = 50(\text{kA}),\ \delta_0 = 0{,}25,\ \omega = 17{,}9 \cdot 10^3 \left(\text{s}^{-1}\right)$, the pulse length is 0.001(s). At the first stage of solution, space-time distributions of vector components of EM-field were obtained, then deformation was analyzed. Fig. 2 shows a graph of the time-dependent movements of the center of the workpiece (this is the point that is most exposed to EM-field in this case). It can be seen from the graph that damping oscillations occur, and they continue longer than the current pulse. This fact should be taken into account when creating technological systems of a similar class. Figure 3 shows the deformed state of the system, which corresponds to the maximum displacements.

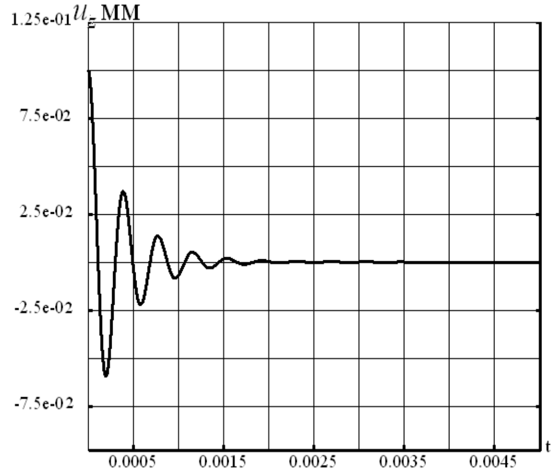

Fig. 2 – Change in time of displacements of the workpiece center

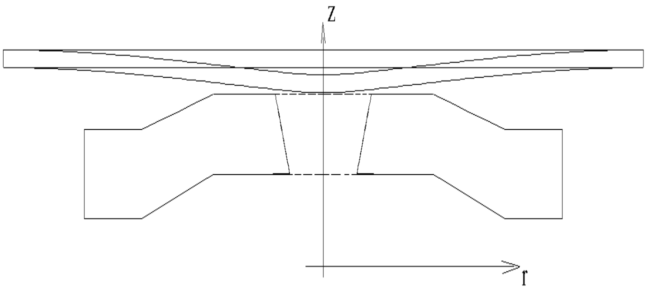

Fig. 3 – Deformed state

Conclusions. The article formulates an actual scientific and practical problem of non-stationary deformation of structural elements under the action of an electromagnetic field, presents a mathematical statement of the problem and gives an example of a solution.

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