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OSCILLATIONS OF BEAMS IN TENSION UNDER CREEP CONDITIONS

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The paper is devoted to the approach and analytical methods of predicting changes in the eigen oscillations of beams under the conditions of longitudinal tension during creep. On the basis of the classical equation of oscillations of beams under tension, a method for estimating eigen frequencies that can vary during creep is obtained. Expressions for the longitudinal force, which depends on the physical and mechanical parameters of the material, were obtained. A relationship was found for determining the time of significant influence of creep on the eigen frequencies. With the help of the obtained expression for the value of the time to fracture of the beam using the Kachanov continuity parameter, an approach to determining the influence on the frequency of the hidden damage accumulation process is proposed. The case of large deflections of the beam is considered in a geometrically nonlinear statement using the method of many scales with the expansion of the solution by a small parameter. The processes of dynamic creep are considered, in which the acceleration of the rate of creep strains in the material is provided by the contribution of amplitude stresses. The resulting equation is solved by the method of weighted residuals in the Galerkin form. Dependencies for determining the rate of stress relaxation in the beam during creep were obtained. The limiting values of the compressive force in terms of the loss of stability of the beam under the given conditions of creep and oscillations are estimated. Computational modeling was performed and results were obtained that allow determining the sensitivity of the eigen oscillation frequencies of beams made of different structural materials to tensile creep. Heat-resistant alloys, alloyed steels, titanium, aluminum alloys and tin-lead solder at temperatures inherent in the typical operating conditions of structural elements made from them are considered. It is shown that the smallest effect of creep on eigen frequencies is found for light alloys. With the help of the obtained ratios, the frequencies of nonlinear oscillations that can occur in the beams were analyzed, and a skeletal curve was built.

Ключові слова: oscillations, eigen frequencies, creep, beam, tension, bending.

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КОЛИВАННЯ БАЛОК ПРИ РОЗТЯЗІ В УМОВАХ ПОВЗУЧОСТІ

Статтю присвячено підходу та аналітичним методам прогнозування змінювання власних коливань балок в умовах дії сил поздовжнього розтягу при повзучості. На базі класичного рівняння коливань балок при розтязі отримано метод оцінювання власних частот, які можуть змінюватись при повзучості. Отримано вирази для поздовжньої сили, що залежить від фізико-механічних параметрів матеріалу. Знайдено співвідношення для визначення часу істотного впливу повзучості на власні частоти. За допомогою отриманого виразу для значення часу до руйнування балки з використанням параметру суцільності Качанова запропоновано підхід до визначення впливу на частоти процесу накопичення прихованих пошкоджень. Випадок великих прогинів балки розглянуто у геометрично нелінійній постановці з використанням методу багатьох масштабів з розвиненням розв'язку за малим параметром. Розглянуто процеси динамічної повзучості, в якій прискорення швидкості деформацій повзучості в матеріалі забезпечується внеском амплітудних напружень. Розв'язок отриманого рівняння виконується методом зважених відхилів у формі Гальоркіна. Отримано залежності для визначення швидкості релаксації напружень у балці при повзучості. Оцінено граничні значення сили стискання з точки зору втрати стійкості стрижня в заданих умовах повзучості та можливості виникнення коливань. Виконано розрахункове моделювання та отримано результати, що дозволяють визначити чутливість частот власних коливань балок, виготовлених з різних конструктивних матеріалів, до розтягу при повзучості. Розглянуто жароміцні сплави, леговані сталі, титановий, алюмінієвий сплави та олов'яно-свинцевий припій при температурах, які притаманні типовим експлуатаційним умовам конструктивних елементів, що з них виготовляються. Показано, що найменший вплив повзучості на власні частоти виявлено для легких сплавів. За допомогою отриманих співвідношень проаналізовано частоти нелінійних коливань, що можуть відбуватись у балках, та побудовано скелетну криву.

Ключові слова: коливання, власні частоти, повзучість, балка, розтяг, згин.

Introduction. In many engineering applications, beam elements, which can be irreversibly deformed due to the occurrence of creep deformation under conditions of high temperature effects, are widely used. Models of rods (tension) and beams (bending) are also often used as a design model of a structural element. Cyclic changes in the deformed state, or oscillations during irreversible deformation, occur in structures of power and transport engineering [1], cable engineering [2], etc.

Methods of estimating frequencies and modes of oscillations of beam systems are well developed and implemented both analytically and numerically [3-5]. However, taking into account the effect of static loading is a more difficult problem. Since the appearance of the elastic classical formulation of the problem by J. Morrow [6] and S.P. Timoshenko [7], this problem continues to remain in the center of attention of researchers [2, 4, 8-10]. The approaches of the Euler–Bernoulli beam theory [8, 9] or the theory of S.P. Timoshenko [7] are used. It is noted that if the initial tension of the beams by axial forces is not extremely large, even relatively small transverse amplitudes of oscillations can cause significant nonlinear effects [4]. Static and dynamic approaches for evaluating the influence of fastenings on the values of axial forces have also been formulated [9, 10].

On the other hand, the irreversible deformation of the beam material, which occurs over time in beams during creep, was studied separately. Approaches and methods for calculating beams that bend and tense during creep are well developed [11, 12]. The authors of [13] note that the solutions of such problems are necessary for a detailed analysis of the stress-strain state, because when polycrystalline materials are deformed at elevated temperatures with moderate values of the applied forces, significant stresses occur in some places. Models are being developed for estimating beam failure due to creep [14].

Analysis of the real behavior of structural elements under conditions of high temperature and cyclic loading, which is implemented, for example, in turbine blades, shows that the processes of oscillations and irreversible deformation during creep occur simultaneously. In this regard, the methods for calculating the stress-strain state that must simultaneously take into account both processes have to be developed. The methods for the evaluation the influence of vibrations on the creep and the damage accumulation in the material of the structural element were developed in previous authors' papers [15, 16]. The study of the inverse problem, the influence of creep on the amplitudes of forced oscillations was started in [17]. This publication continues research in this direction. The eigen oscillations of the beams are studied, taking into account their pre-tension under creep conditions. **Eigen oscillations of beams with consideration of creep due to preliminary tension.** Eigen and free oscillations of the beams are studied, taking into account their preliminary tension and creep due to this tension. If the nature of natural oscillations of a structure is known, then it is possible to evaluate its inherent internal properties that manifest themselves under the action of external disturbances.

In the case of eigenoscillations of systems with constant stiffness, the amplitudes of the deflections of the system points from the equilibrium position do not depend on the frequency and, when oscillations occur, are determined only by the initial conditions. In the case of small linear oscillations, the stiffness characteristics can be considered constant and the internal forces are reduced to bending stresses. If, at the same time, preliminary tension (compression) is taken into account, then in addition to the shear force from bending, the equations of motion of the element include a term with transverse force in the form of a projection of membrane stresses on the normal to the axis, and the equations of motion have the following form [7]:

$$EIw^{IV} \mp Pw'' + \rho\delta\ddot{w} = 0\,, \qquad (1)$$

where EI is beam bending stiffness; P is membrane force; sign «–» correspond to tension as well as «+» for compression; $\rho\delta$ is beam linear mass; $(\ )' = \partial(\ )/\partial x$, $\dot{(\ )} = \partial(\ )/\partial t$. Equation (1) should be supplemented with boundary and initial conditions. For example, boundary conditions for hinged support: $w(0,t)=w''(0,t)=w(L,t)=w''(L,t)=0$, L is beam's length- Initial conditions: $w(x,0)= L_1(x)$, $\dot{w}(x,0) = L_2(x)$.

When deriving (1), it is assumed that the level of preloading is sufficiently high and the influence of bending stresses on it can be neglected. Under these conditions, due to creep, the membrane force, the level of which during preliminary tension (compression) is equal to P_0, will change during holding (relax). Consequently, creep will affect the eigen oscillation characteristics, since the previously found value of the membrane forces P is included in the initial conditions. Besides, these above conditions include the value of $P=P(\tau)$, where τ is the holding time of the beam during creep to the moment of occurrence of transverse vibrations.

If oscillations occur after holding at creep, then

$$P = P_0\left[1+(m-1)E\delta^{1-m}P_0^{m-1}\Omega(\tau)\right]^{-\frac{1}{m-1}}, \qquad (2)$$

where P_0 is absolute value of preliminary loading of a beam; τ is creep holding time before to oscillations occurrence; m, $\Omega(\tau)$ are creep material parameters, which are determined by creep curves data processing by use of Norton law [11].

Fort the secondary creep its rate is determined by the following relations:

$$\dot{c} = B\sigma^m\,,\ c(0) = 0\,,\ , \qquad (3)$$

$$\Omega(t) = \int_0^t B(t)\,dt = B_1 t\,, \qquad (4)$$

$$\sigma = P_0 / \delta\,.$$

The circular frequency of transverse oscillations of the beam is determined from (1) and, for example, for fixed hinged edges

$$\omega_n = \omega_n^0\sqrt{1 \mp P/P_{*n}}\,, \qquad (5)$$

where ω_n^0 is angular eigen frequency of beam transverse oscillations without preliminary tension (compression); $P_{*n} = \left(\frac{n\pi}{L}\right)^2 EI$ is Euler's critical force at buckling in *n-th* form [7]; P is the value of load, obtained by use of (2). In the case of compression with load value P_{*n} the beam loses its stability and $\omega_n = 0$.

Let us introduce the following notation and take into account (4):

$$\varphi_0 = B_1(m-1)\left(\frac{P_0}{\delta}\right)^{m-1} E\,, \qquad (6)$$

$$\chi(\tau) = \frac{1}{\sqrt[m-1]{1+\varphi_0\tau}}\,. \qquad (7)$$

Then

$$P = P_0\chi(\tau)\,, \qquad (8)$$

$$\omega_n = \omega_n^0 \sqrt{1 \mp \frac{P_0}{P_{*n}} \chi(\tau)} \; . \qquad (9)$$

It follows from the last relation (9) that the angular frequency of beam oscillations that occurs after creep exposure by preliminary tension (minus in the expression under the root in (9)) or compressed (plus - in (9)) will depend on the duration of creep until the moment of oscillations occurrence τ. Moreover, since $\chi(\tau)$ decreases with increasing τ, we can conclude that the effect of preliminary tension (compression) under creep conditions decreases with holding, and theoretically at $\tau \to \infty$, the angular frequency will tend to the frequency of eigen vibrations without taking into account creep. Note that if the relaxation rate is low, then a sufficiently high level of residual stresses is established in the beam, which persists for a long time, and in this case creep fracture can occurs. In this case, $\chi(\tau) \cong \text{const}$ up to fracture moment.

The oscillations that arise during this period will depend only on the initial deviations and their rate, while the membrane forces practically do not vary. Following this, the creep time from the point of view of the oscillations properties can be divided into two segments $(0, \tau_{st})$ and (τ_{st}, τ_{fr}). In the first segment, the angular oscillation frequency depends on the exposure time $\tau \in (0, \tau_{st})$, and in the second isn't. However, in the second case, fracture due to creep is possible.

Let us analyze the nature of the change in the function $\chi(\tau)$ (7) included in (9). Consider several materials used in conditions where creep properties cannot be avoided - high-temperature alloys, S816 and N155, alloy steels SUS 347 and 45X14N14V2M, titanium alloy OT-4, aluminum alloys - duralumin D16AT and cast alloy AMn6, solder 81Pb-19Sn. The physical and mechanical properties of these materials are presented in the Table at the operating temperatures of the structural elements from which they are made. The calculated dependencies $\chi(\tau)$ are shown in Fig. 1, the curve numbers in the figure correspond to the row numbers in the table.

The results obtained show that aluminum alloys and solder have the best properties in terms of the rate of return of the oscillation frequency to its own values, followed by a titanium alloy. The effect of tension on heat-resistant alloys continues for a long time. Alloy steels show different behavior.

Unfortunately, it is not possible to evaluate the properties of the alloy response to tension during creep by the value of one parameter: it is known that the processing of creep curves for the same stress range can give different combinations of the factor *B* and the exponent *m*. In the general case, the use of relation (7) can be recommended.

Number of curve	Material	Tempe-rature, K	Young modulus $E10^{-5}$, MPa	Creep constant B , МПа$^{-m}$/h	Creep constant m
1	S816	1089	1.55	$1.68{\cdot}10^{-23}$	9.17
2	N155	1025	1.58	$2.79{\cdot}10^{-20}$	6.68
3	SUS 347	923	1.6	$7.6{\cdot}10^{-14}$	3.7
4	45X14N14 V2M	873	1.65	$2{\cdot}10^{-10}$	3
5	OT-4	773	1.0	$1.78{\cdot}10^{-36}$	18.2
6	D16AT	593	0.33	$3.39{\cdot}10^{-8}$	3
7	AMn6	593	0.7	$6.16{\cdot}10^{-9}$	4.5
8	81Pb-19Sn	293	0.55	$6.83{\cdot}10^{-7}$	3

Table 1 – Material constants

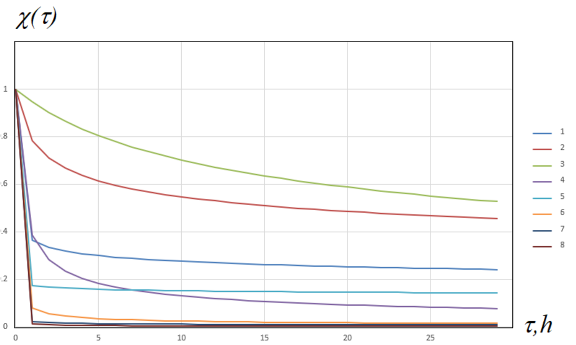

Fig.1 – Dependence between influence function $\chi(\tau)$ and holding time τ

Under relaxation conditions, when the stress continuously decreases, it does not lead to discontinuity. The relaxation rate turns out to be significant, which depends on the physical nature of the materials, the level of initial stresses and temperature, as well as on other external factors. In essence, under conditions of continuous relaxation, ordinary aging occurs, and the residual stress rather quickly becomes less than safe for fracture, and the oscillation frequency is determined by the holding time of the rod during creep.

Analysis of creep fracture time.

Let us concentrate on determining the time before fracture of the rod, taking the fracture model with the kinetic equation for the Kachanov‘s continuity parameter ψ [18] in the following form:

$$\dot{\psi} = -A_1 \left[\frac{\sigma(t)}{\psi} \right]^r , \qquad (10)$$

where $\psi(0)=1$; $\psi(\tau_r)=0$; A_1, r are material constants, which are usually determined from creep and long term curves.

From (8) we have:

$$\sigma(t) = D_m \left[\tau + 1/\varphi_0 \right]^{\frac{1}{1-m}} , \qquad (11)$$

where D_m is the constant, depending on temperature and material properties, since the material constants of elasticity and creep depend on them:

$$D_m = \left[(m-1) B_1 E \right]^{\frac{1}{1-m}} . \qquad (12)$$

Here φ_0 is the initial stress function, as follows from (6).

Substitution (11) into the evolution equation (10) and subsequent integration leads to the dependence

$$\tau_r = \left[B_{mr} + \varphi_0^{-k} \right]^{\frac{1}{k}} - \varphi_0^{-1}, \quad (13)$$

where

$$B_{mr} = \frac{k\left[(m-1) B_1 E\right]^{\frac{1}{m-1}}}{A_1 (r+1)}, \; k = \frac{m-r-1}{m-1}. \quad (14)$$

For the case of material with linear visco-elastic properties $\dot{c} = B\sigma$ and in can be found from Maxwell equation [19]:

$$P = P_0 e^{-\frac{\tau}{t_r}}, \quad (15)$$

where $t_r = -\frac{B}{E}$ is relaxation time value.

Correspondingly, for the fracture time value we find

$$\tau_r = -\frac{t_r}{r} \ln\left(1 - r\frac{\tau_\sigma}{t_r}\right), \quad (16)$$

where $\tau_\sigma = \frac{1}{A_1 \left(\frac{P_0}{\delta}\right)^r (r+1)}$ is the fracture time value at stress $\sigma_0 = \frac{P_0}{\delta}$ in creep conditions.

The angular frequency of oscillations that occur after holding, in the case of linear viscoelasticity of the material, will be calculated as follows:

$$\omega_n = \omega_n^0 \sqrt{1 \mp \frac{P_0}{P_{*n}} e^{-\frac{\tau}{t_r}}}, \; \tau \in (0, \tau_r), \quad (17)$$

where τ_r is calculated by use of Eq.(16) and determines the value of the time to fracture of the beam during relaxation.

Viscoelastic behavior is characteristic of polymers at $\sigma_0 = \frac{P_0}{\delta} \le (0.5 \div 0.6)\sigma_T$, where σ_T is the yield limit of material. In the area of nonlinear creep $\sigma_0 \ge 0.5\sigma_T$, $m >> 1$, and $\varphi_0^{-1} \approx 0$ approximately can be taken. This supposition is equivalent to the assumption that the change in membrane stresses after a long exposure is independent of the initial loading. Then

$$\chi(\tau) = D_m \tau^{\frac{1}{1-m}} / \sigma_0,$$

$$\tau_r = \sqrt[k]{B_{mr}},$$

$$\omega_n = \omega_n^0 \sqrt{1 \mp \frac{\delta}{P_{*n}\left[(m-1) B_1 E \tau\right]^{\frac{1}{m-1}}}}. \quad (18)$$

Consideration the influence of transverse deflections on the magnitude of the membrane force. Let us consider the previous problem, but taking into account the effect of transverse deflection's influence on the value of the membrane force. First, for large deflections, it is necessary to take into account the additionally occurring membrane force *ΔP*. Secondly, we take into account the periodicity of the membrane force in the case of arising oscillations, which intensifies the creep of the rod in the axial direction. We investigate the influence of the latter on the eigen transverse oscillations of the rod.

Let the beam be preliminarily tensed by force P_0, then its elastic initial strain and displacement are

$$\varepsilon_0 = \frac{P_0}{E\delta}, \; u_0 = \frac{P_0 L_0}{E\delta}. \quad (19)$$

We assume that the edges of the beam in tension are further fixed. When held under creep conditions, the membrane force will relax, and its value during the holding time τ is calculated by Eq.(8). When oscillations occur, the strain of the beam neutral axis will change

$$\varepsilon = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad (20)$$

where u, w are the displacements of points of the neutral axis in the axial and transverse directions when the rod is bent.

The closing of the beam edges, taking into account (20), will be written as an equality:

$$\lambda = \int_0^L \frac{\partial u}{\partial x} dx = -\int_0^L \varepsilon_x dx + \frac{1}{2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx.$$

And since the edges do not close each other

$$\int_0^L \varepsilon_x dx = \frac{1}{2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx. \quad (21)$$

The resulting equation shows that additional strains at bending are determined by the deflections of the beam axis during its oscillations. Consequently, a material of tensed beam, when oscillations, will run due dynamic creep mode [19]. The dynamic component of the membrane force can be determined from (21).

Let us compose the equation of bending oscillations:

$$EIw^{IV} - \sigma\delta w'' + \rho\delta\ddot{w} = 0, \quad (22)$$

where $\sigma\delta$ is membrane force, which can be represented as an asymptotic expansion in a small parameter $\mu = 2\pi/\Omega$, where Ω is a frequency of these oscillations in the direction of beam axis:

$$\sigma\delta = \sigma^0(t)\delta + \mu\sigma^1(\xi)\delta, \quad (23)$$

and $\sigma(x;t;\xi)$ depends upon two variables t and $\xi = \frac{t}{\mu}$, so, that $\sigma^0(t)$ is slow varying coefficient in expansions (23), as well as $\sigma^1(x,\xi)$ is fast varying stress.

For axial strain during creep, let us write the governing equation, taking into account elastic strain:

$$\varepsilon_x = \frac{\sigma}{E} + \int_0^t \dot{c}(\sigma)\,dt\,, \tag{24}$$

where $c(\sigma)$ are creep strains, which connected with stresses by Norton law:

$$\dot{c} = B\sigma^m,\ m>1.$$

From (24) considering (23) we obtain:

$$\varepsilon_x^0(t) = \frac{\sigma^0}{E} + B_1\frac{t-\tau}{T}\int_0^T \left(\sigma^0(t) + \mu\sigma^1(\xi)\right)^m d\xi\,, \tag{25}$$

$$\varepsilon_x' = \frac{\sigma'}{E}\,.$$

At the same time, we note that the average axial strain of the beam axis does not change with time and remains equal to ε_0, determined by Eq. (19):

$$\frac{1}{L}\int_0^L \varepsilon_x^0(t)\,dx = \varepsilon_0\,,$$

which, taking into account the first equation from (23), allows us to write the relaxation equation for cyclic deformation under creep conditions as follows:

$$\frac{d\sigma^0}{dt} = -B_1\frac{E}{T}\int_0^T \left(\sigma^0(t) + \mu\sigma^1(\xi)\right)^m d\xi\,, \tag{26}$$

$$\sigma^0(0) = \sigma(\tau)\,,$$

where $\sigma(\tau) = \frac{P(\tau)}{\delta}$, $P(\tau)$ is calculated due to (8) and is equal to the value of stresses in the beam before oscillations start.

However, from (21) we find, using the second equality from (25):

$$\mu\frac{\bar{\sigma}^1}{E} = \frac{1}{2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\,. \tag{27}$$

Here $w=w(x,\xi)$ are deflections of the beam axis that are fast varied with time; $\bar{\sigma}^1 = \int_0^L \sigma^1(x,\xi)\,dx$ is definite average stress that occurs in the beam when it oscillates.

Let us write the beam oscillation equation in the form

$$EIw^{IV} - \bar{\sigma}\delta w'' + \rho\delta w_{,\xi\xi} = 0\,, \tag{28}$$

where $w_{,\xi\xi} = \frac{d^2 w}{d\xi^2}$, $\bar{\sigma} = \sigma^0 + \mu\bar{\sigma}^1(\xi)$.

If the beam is hinged, then we represent its deflections during oscillations in the form

$$w(x,\xi) = f(\xi)\sin\frac{\pi}{L}x\,.$$

Next, we use the Galerkin method [20] to solve equation (28). Let us first take into account that from (27) it follows

$$\mu\bar{\sigma}^1 = \frac{\pi^2 E}{4L^2} f^2(\xi)\,,$$

and

$$\bar{\sigma} = \sigma^0 + \frac{\pi^2 E}{4L^2} f^2(\xi)\,. \tag{29}$$

By use Galerkin method for Eq.(28), we obtain:

$$\frac{d^2 f}{d\xi^2} + \omega_o^2\left(1 + \frac{P(t)}{P_*} + Kf^2\right) f = 0\,, \tag{30}$$

where $\omega_0 = \left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{\rho\delta}}$ is angular frequency of small oscillations of the beam without preliminary tension; $P(t) = \bar{\sigma}^0\delta$ is membrane force, which is determined from the solution of Eq. (26), which describes stress relaxation during dynamic creep; $P_* = EI\left(\frac{\pi}{L}\right)^2$ is Euler critical force for a rod in compression; $K = \frac{1}{4}\frac{\delta}{I}$.

Let us consider the solution of Eq. (30) in the form

$$f = A\cos\omega\xi\,,$$

where A is amplitude and ω is angular frequency of oscillations.

Let us integrate equation (30) over the total oscillation period $T = 2\pi/\omega$, by use Galerkin method. We consider, that

$$\int_0^{2\pi/\omega} \cos^2\omega\xi\,d\xi = \frac{\pi}{\omega}\,,\quad \int_0^{2\pi/\omega} \cos^4\omega\xi\,d\xi = \frac{3\pi}{\omega}\,.$$

We obtain

$$\int_0^{2\pi/\omega}\left(\frac{d^2 f}{d\xi^2} + \omega_o^2\left(1 + \frac{P(t)}{P_*} + kf^2\right) f\right)\cos\omega\xi\,d\xi =$$

$$= A\left(\omega_o^2\left(1 + \frac{P(t)}{P_*}\right) - \omega^2\right)\frac{\pi}{\omega} + \frac{3}{4}KA^3\frac{\pi}{\omega}\omega_o^2 = 0.$$

As a result, we have a dependence between the frequency of nonlinear oscillations ω and the amplitude A, which, under creep conditions, depends parametrically on time:

$$\omega^2 = \omega_o^2\left(1 + \frac{3}{4}KA^2 + \frac{P(t)}{P_*}\right). \tag{31}$$

We introduce the notation for the ratio of the value ω to the corresponding frequency of linear oscillations ω_0:

$$\nu^2 = \left(\frac{\omega}{\omega_0}\right)^2 = 1 + \frac{3}{4}KA^2 + \frac{P(t)}{P_*}\,.$$

In coordinates ν, A we obtain a hard-type skeletal curve for a fixed time t (Fig. 2).

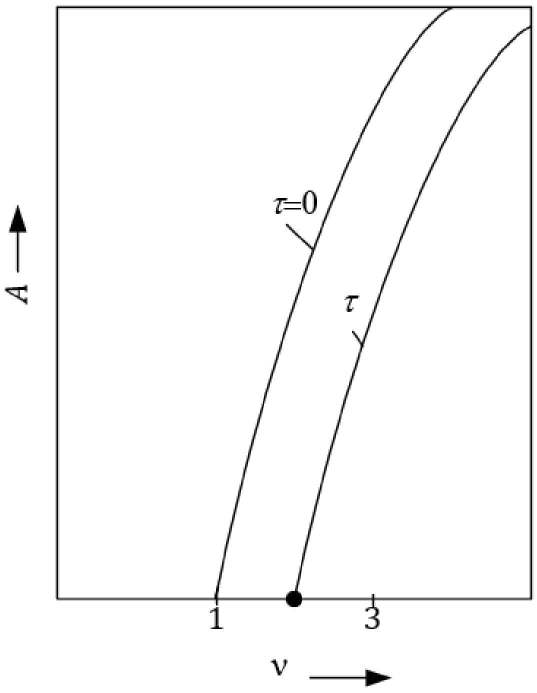

Fig. 2 – Sceletal curve

Let us transform Eq.(26) taking into account the relations found above:

$$\mu\sigma^{1}\left(\xi\right)=\frac{\pi^{2}E}{4L^{2}}A^{2}\cos^{2}\omega\xi,\ 0\le\xi\le 1,\ ,$$

$$\frac{d\sigma^{0}}{dt}=-B_{1}\left(\sigma^{0}\right)^{n}\frac{E}{2\pi}\int_{0}^{2\pi}\left(1+\frac{\pi^{2}E}{4L^{2}}\frac{A^{2}}{\sigma^{0}}\cos^{2}\omega\xi\right)^{n}d\left(\omega\xi\right),$$

$$\sigma^{0}\left(0\right)=\sigma^{0}\left(\tau\right)$$

The obtained relations determine the rate of relaxation.

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Conclusions. The paper presents approaches to solving the problems of beam dynamics and deformation in a new formulation, which includes consideration of preliminary tension. Analytical dependences have been obtained, which allow determining the influence of tensile forces on oscillation frequencies, time to failure, and relaxation rate. Dependencies between the creep characteristics of the material and the time in which creep holding influences the frequency values were established. A skeletal curve of nonlinear oscillations during creep is obtained.

The resulting dependencies can be used in the assessment of dynamics, deformation and strength of structural elements described by beam calculation schemes.

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