UDC 62-253.5 DOI:10.20998/2078-9130.2021.2.245504

K. V. SAVCHENKO, A.P. ZINKOVSKYI, YE. O. ONYSHCHENKO, V. O. KRUTS, S. M. KABANNYK

DIAGNOSTICS OF THE PRESENCE OF A FATIGUE CRACK IN THE COMPRESSOR BLADE USING THE SPECTRUM OF VIBRATION AMPLITUDES AT THE SUPERHARMONIC RESONANCE

In this study, the solution of the forced vibration response of a structural element with a fatigue crack was carried out using the finite element method to determine the influence of its presence on the flexural forced vibration behaviour of the compressor blade airfoil at the superharmonic resonance of the order 1/2. The blade airfoil with a low twisted angle was used as an object of investigation to perform the computational analysis. Its vibrations in the plane of minimum stiffness were excited by the kinematic displacement of root edge elements. The fatigue crack was modelled as a mathematical cut. Two locations of the crack were investigated – on the leading edge and convex side of the blade airfoil. The nonlinearity due to the intermittent contact of the crack surfaces, which is caused by the opening and closing of the crack during each vibration cycle, was taken into account by solving the contact problem. To quantify this kind of nonlinear dynamic behaviour, the vibration diagnostic parameter was defined as the displacement amplitude ratio of the dominant harmonics at the superharmonic resonance of the order 1/2. Based on the results of the calculations it has been found that regardless of the crack location, the ratio nature is the same for all vibration axes. However, with vibrations in the plane of minimum stiffness, the crack on the convex side of the airfoil has an opening mode propagation, which makes it possible to fix its location due to a sharp change in the ratio of the amplitudes of the dominant harmonics along the corresponding axis.

Keywords: compressor blade airfoil, breathing crack, forced vibration, superharmonic resonance, vibration diagnostic parameter.

В роботі представлено результати чисельних досліджень на базі скінченноелементного моделювання вимушених коливань конструктивного елемента з тріщиною втоми, та визначено вплив її наявності на вимушені згинні коливання пера лопатки компресора при супергармонічному резонансі порядку 1/2. В якості об'єкту дослідження розглядається перо лопатки компресора з малим кутом закрутки. Його коливання у площині мінімальної жорсткості збуджувались кінематичним переміщенням торцевих елементів пера. Тріщина втоми моделювалась математичним розрізом. Було розглянуто два місцеположення тріщини – на вхідній кромці та на спинці пера лопатки. Непостійність контакту берегів тріщини, обумовлена її почерговим відкриттям і закриттям викликає нелінійність, яка була врахована шляхом розв’язання контактної задачі. Для оцінки нелінійної динамічної поведінки пера лопатки використовується вібродіагностичний параметр, а саме, відношення амплітуд переміщень домінуючих гармонік при супергармонічному резонансі порядку 1/2. За результатами досліджень встановлено, що, незалежно від місцеположення тріщини, характер зазначеного відношення однаковий для всіх осей коливань пера лопатки. При цьому, при коливаннях у площині мінімальної жорсткості, спостерігається «дихання» тріщини втоми на спинці пера, що дозволяє зафіксувати її місцеположення, яке відповідає різкій зміні відношення амплітуд домінуючих гармонік переміщень уздовж відповідної осі.

Ключові слова: перо лопатки компресора, дихаюча тріщина, вимушені коливання, супергармонічний резонанс, вібродіагностичний переметр.

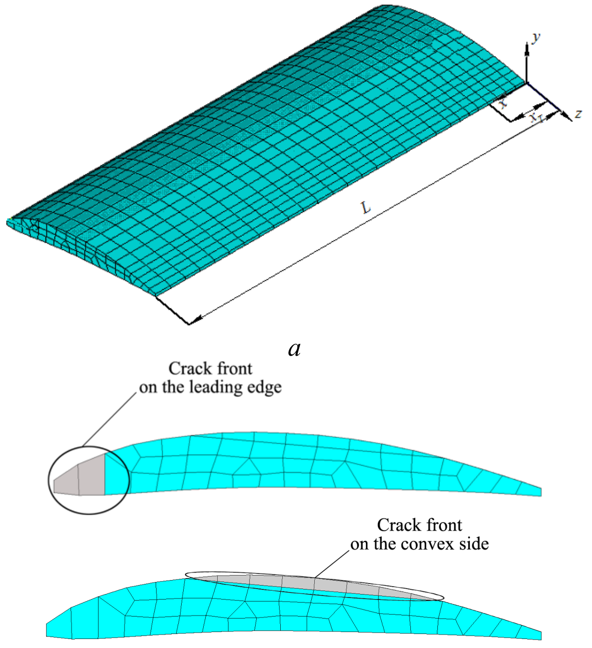
В работе представлено результаты численных исследований вынужденных колебаний упругого конструкционного элемента с усталостной трещиной, выполненное с использованием метода конечных элементов, для определения влияния ее наличия на вынужденные изгибные колебаниях пера лопатки компрессора при супергармоническом резонансе порядка 1/2. В качестве объекта исследования для проведения анализа было использовано перо лопатки компрессора с малым углом закрутки. Его колебания в плоскости минимальной жесткости возбуждались кинематическим перемещением торцевых элементов. Усталостная трещина моделировалась математическим разрезом. Было исследовано два местоположения трещины - на входной кромке и на спинке пера лопатки. Нелинейность из-за прерывистого контакта берегов трещины, вызванного ее поочередным открытием и закрытием во время каждого цикла колебаний, была учтена при решении контактной задачи. Для оценки такого нелинейного динамического поведения, был определен вибродиагностический параметр как отношение амплитуд перемещений доминирующих гармоник при супергармоническом резонансе порядка 1/2. По результатам расчетов установлено, что, независимо от местоположения трещины, характер указанного отношения одинаков для всех осей колебаний пера лопатки. Однако, при колебаниях в плоскости минимальной жесткости, наблюдается «дыхание» усталостной трещины на спинке пера, что позволяет зафиксировать ее местоположение за счет резкого изменения соотношения амплитуд перемещений доминирующих гармоник вдоль соответствующей оси.

Ключевые слова: перо лопатки компрессора, дышащая трещина, вынужденные колебания, супергармонический резонанс, вибродиагностический параметр.

Introduction. The presence of fatigue cracks in the complex structural system such as turbomachine may lead to dangerous effects for its safety and cause abnormal behaviour. Therefore, vibration analysis of structures with cracks operating under dynamic loading conditions is an important area of research due to the growing demands for reliable damage detection techniques. The presence of such damage not only causes a local change in the mechanical characteristics of the structure at its location, but it also affects in general on the structure [1]. Thus methods based on the dynamic behaviour of structures with fatigue cracks can be an effective means for damage detection in non-destructive tests.

Open crack and breathing crack models are the two main categories of crack models used in vibration-based detection methods. The first models assume that the crack in a structural element is open and remains open during vibration. The assumption is usually satisfied when the damage is rather large and avoids the difficulties resulting from the nonlinear behaviour of breathing crack. On the other hand, the breathing crack model is generally used in the case the damage occupies only a small portion of the cross-section of the structural element. In this case, it requires a nonlinear model to take into account its effect on the dynamic behaviour of the system. Consequently, vibration-based methods are also classified into two categories: linear and nonlinear approaches.

In the linear approach, the presence of damage in the structure is detected by monitoring the changes in modal parameters, such as natural frequency, and mode shape [2, 3]. Due to the low sensitivity of these indicators they usually require a rather large size of the damage in the system to provide an adequate identification [4].



a

b

Fig. 1 – The FE model of the airfoil of the investigated compressor blade (*a*) and its cross-section areas with crack (*b*)

In the nonlinear approach, the breathing crack model considers that, during the vibration cycle, the edges of the crack come into and out of contact, leading to changes in the dynamic behaviour of the structure, which might be useful for the detection of cracks. In these methods, characteristics of the nonlinear dynamic response such as the presence of sub and super harmonics or changes in the phase diagrams can be used as damage indicators [1, 5]. As mentioned earlier, the excitation of the structure with fatigue crack by a harmonic force causes "breathing" of the crack which leads to appearance harmonics that are integer multiple or fractional multiple of the forcing frequency. These harmonics are commonly named as sub and super harmonics, respectively. These features are easily detectable and more sensitive to characteristics of the damages in comparison with the modal properties of a linear system. However, the researchers' attention has been focused mainly on the investigations of the nonlinear effects caused by the presence of breathing cracks in the rods with different cross-section types [1, 5 - 9].

Therefore, the main goal of the present paper is to determine the vibration diagnostic parameter, as the displacement amplitude ratio of the dominant harmonics at the superharmonic resonance of the order 1/2, indicating the presence of a breathing crack on the leading edge and convex side of the blade airfoil.

Investigation object and its modelling. The gas-turbine engine low-pressure compressor blade was chosen as the object of investigation for computational experiments. Its airfoil with a low twisted angle has length L = 105 mm and is made of BT-2 titanium alloy with the following physical and mechanical characteristics: modulus of elasticity of the 1st kind $E = 1.15 \cdot 10^{11}$ Pa; density $\rho = 4500$ kg/m³; Poisson's ratio $\mu = 0.3$.

In the study, a finite element method (FEM) is used. As in [9], for FE modelling of the blade airfoil under investigation with allowance for its design features, we divided it into portions. In that case, a linear eight-node finite element and its modifications were chosen for modelling. Previously they were used in solving similar problems for damaged blades, and rods of a rectangular and circular cross-section with fatigue cracks [11, 12]. A general view of the airfoil FE model is shown in Fig. 1*a*. The fatigue damage was simulated as a mathematical cut in two locations (Fig. 1*b*) – on the leading edge and convex side of the blade airfoil at the point x_T = 10.5 mm up from the root cross-section, which is 10% of the airfoil length. The damaged area is 10% of the airfoil cross-section area. The mutual non-penetration of crack faces was ensured by the introduction of the surface-surface contact elements, which makes it possible to take into account different contact conditions.

The forced vibrations of the FE models are described by the nonlinear differential equation:

$$[M]\{\ddot{u}\}+[D]\{\dot{u}\}+[K]\{u\}=\{Q(t)\} \qquad (1)$$

where $[M]$, $[D]$, and $[K]$ are the inertia and dissipation matrices, and the stiffness matrix of the system, respectively, $\{u\}$, $\{\dot{u}\}$, and $\{\ddot{u}\}$ are the column vectors of displacement, velocity, and acceleration, respectively, $\{Q(t)\}$ is the column vector of the external harmonic loading.

Due to the contact interaction of the crack faces during the vibrations, the stiffness matrix changes in time and is defined by the equation:

$$[K]\{u\}-\{\hat{F}\}-\{F^K\}=0 \qquad (2)$$

where $\{\hat{F}\}$ is the vector of nodal forces and $\{F^K\}$ is the vector of nodal forces occurring during the contact interaction of the crack faces.

The nonlinear equation (1) is solved by integrating with time using the Newmark method [13]:

$$\begin{cases} \{\dot{u}\}_{t+\Delta t}=\{\dot{u}\}_t+\left[(1-k_1)\{\ddot{u}\}_t+k_1\{\ddot{u}\}_{t+\Delta t}\right]\Delta t; \\ \{u\}_{t+\Delta t}=\{u\}_t+\{\dot{u}\}_t\Delta t+\left[\left(\frac{1}{2}-k_2\right)\{\ddot{u}\}_t+k_2\{\ddot{u}\}_{t+\Delta t}\right]\Delta t^2 \end{cases} \qquad (3)$$

where k_1 and k_2 are the Newmark integration parameters that determine the accuracy and robustness of the integration.

Investigation Results and Their Analysis. A set of numerical experiments was performed to determine the effect of the fatigue crack presence on the flexural forced vibration behaviour of the blade at the superharmonic resonance of the order 1/2 using the developed FE models of the selected assembly of blades and calculation procedure.

The calculations were performed for the case of a rigid coupling over the root edge elements of the airfoil.

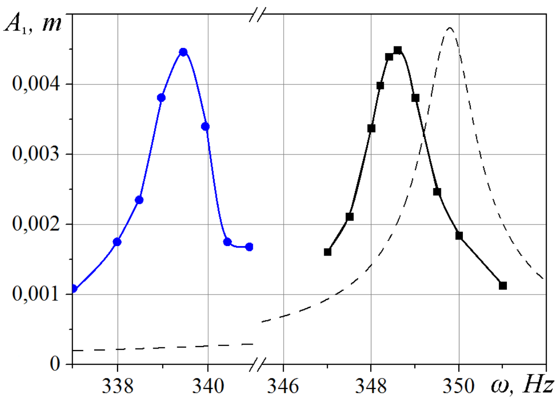


Fig. 2 – Amplitude and frequency characteristics of displacements along with the *Oy* axis of the intact blade airfoil (- - -) and with the breathing crack (● – on the convex side; ■ – on the leading edge) at the fundamental resonance

The forced vibration was excited by kinematic displacement $Q_0\sin(\omega t)$ of the specified elements with an amplitude $Q_0 = 10^{-5}$ m along axis $0y$ at the frequency, which denotes the first flexural mode of vibrations within the plane of its minimum stiffness.

The authors obtained the time dependencies $U_i(t)$ of the free end displacement of the blade airfoil by the Newmark method. Using the results of the harmonic analysis for the portions of these relations characterizing the stationary regime of vibrations, the amplitude and frequency characteristics were derived for the parameters of forced vibrations of the blade airfoil depending on the location of damage (Fig. 2). Here, A_1 –the amplitude value of the first (main) harmonic; ω – excitation frequency. The figure also shows the dependencies for an intact blade airfoil. The data analysis implies the changes in vibration frequency in comparison to the intact airfoil are 0.6% and 3% for the leading edge and convex side respectively, but the level of the amplitudes of the dominant harmonics for the selected locations of damage is the same. This is due to the different influence of the selected damage locations on the stiffness of the blade airfoil. The breathing effect of the damage, which located on the convex side of the airfoil, leads to the change of the airfoil stiffness during its vibration. On the first half-cycle of the vibration, when the damage is closed and its surfaces have full contact, the stiffness of the blade airfoil is equal to an intact one. In the second half-cycle, the damage is open. When the damage located on the leading edge, the contact of crack surfaces characterized by longitudinal shear. Therefore, such type of contact decreases the stiffness less significant, in comparison with the damage on the convex side.

Fig. 3 shows the spectrum of vibration amplitudes of damaged blade airfoil at the fundamental resonance for two locations of damage, where A_i – the amplitude value; i – a harmonic number. As we can see in the spectrum of vibration amplitudes, there are harmonics at frequencies that are multiples of ½, 2, 3, 4, etc. excitation frequency, which corresponds to the fundamental frequency. Also, there is the harmonic of the static component ($i = 0$). From the obtained data it follows that the largest harmonics, after A_1, occur at $i = 0$ and 2, the amplitudes at $i = 1/2$ and 3 are equivalent. The higher harmonics in comparison with the presented are very small and their fixation is problematic.

Next, look into the possibility of excitation of the superharmonic resonance of the order 1/2. Nonlinear resonance is excited at frequency close but not equal to half of the fundamental frequency.

Fig. 4 *a* presents the output signal of the free end of blade airfoil which is nonlinear and causes additional

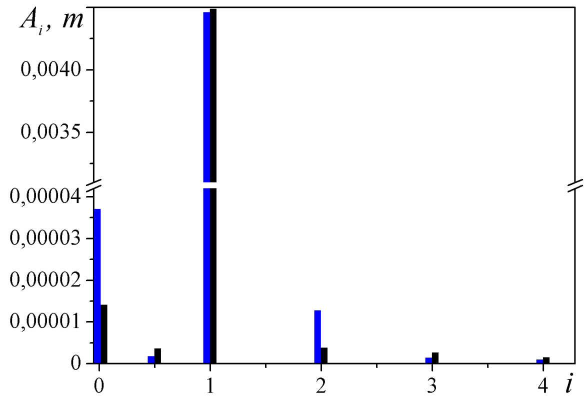
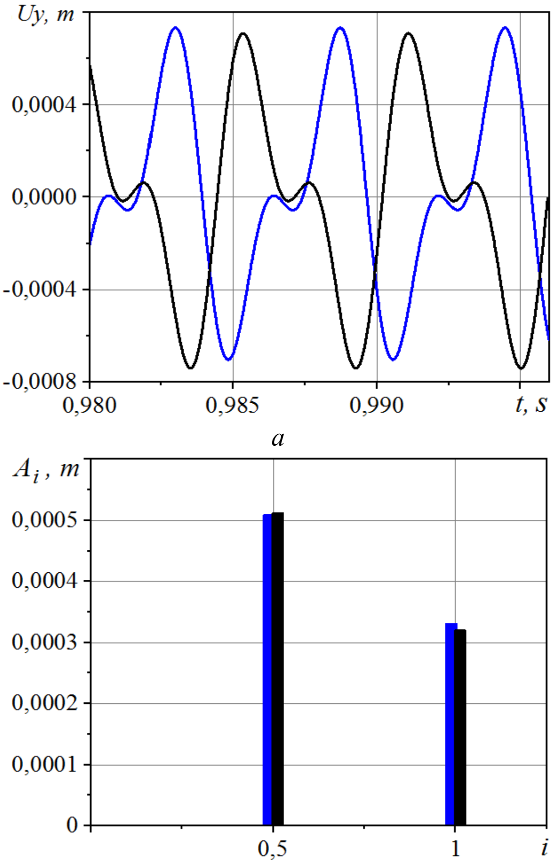


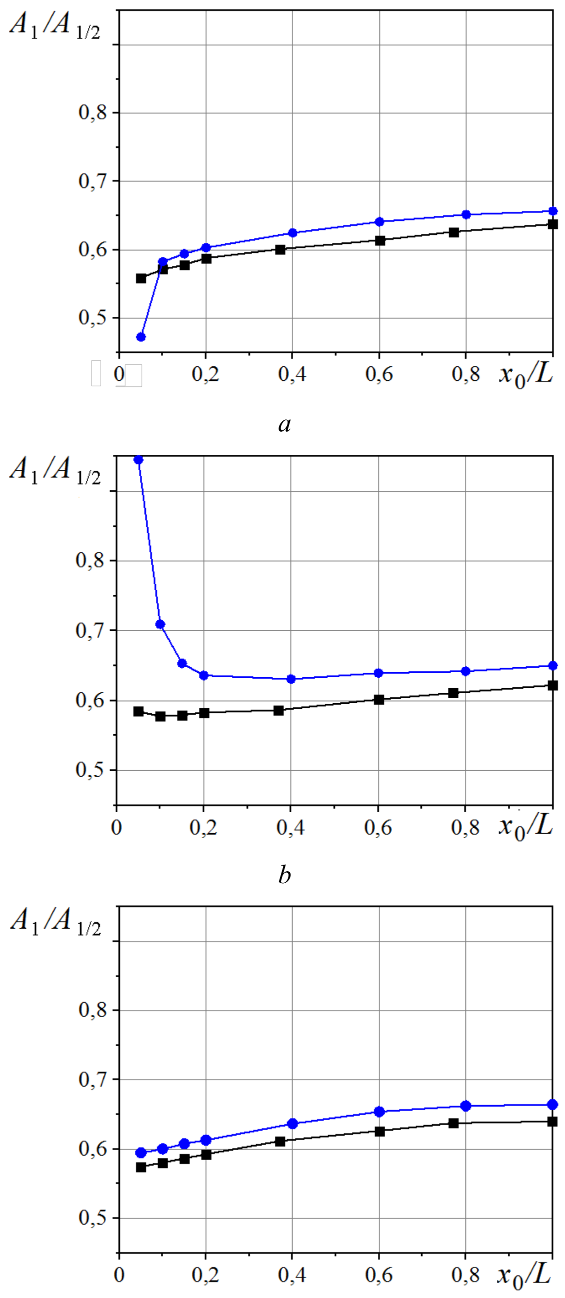
Fig. 3 – The spectrum of vibration amplitudes of blade airfoil with the damage (navy blue line – on the convex side; black line – on the leading edge) at the fundamental resonance



a

b

Fig. 4 – The free end displacements of the blade airfoil with the damage (navy blue line – on the convex side; black line – on the leading edge) (*a*) and corresponding them the spectrum of vibration amplitudes (*b*) at the superharmonic resonance of the order 1/2



a

b

c

Fig. 5 – Diagrams of the amplitudes of the dominant harmonics along with the *Ox* (*a*), *Oy* (*b*) and *Oz* (*c*) axes of the blade airfoil with the damage (● – on the convex side; ■ – on the leading edge) at the superharmonic resonance of the order 1/2 on their registration point

harmonics in the spectrum of its vibrations (Fig. 4 *b*), the largest of which corresponds to the fundamental frequency and has the same order as the harmonics at the excitation frequency, whereas the level of other harmonics a 100 times less.

As mentioned earlier, the possibility of using multiple harmonics due to nonlinear effects caused by the presence of a breathing crack as a vibration diagnostic parameter has recently been the subject of a large number of studies. Moreover, to detect the presence of such type of damage, both their absolute values and ratio may be used. Therefore, in this work, we determined the ratio of the amplitudes of the first harmonic A_1 to the harmonic $A_{1/2}$ at the superharmonic resonance of the order 1/2..

As an example of the obtained results, in Fig. 5 the dependencies of the indicated ratio on the point of registration of vibrations (x_0) along the airfoil length is shown for two locations of the breathing crack. All calculations are presented for the first flexural mode of vibrations.

The analysis of the obtained results shows, regardless of the crack location, their nature is the same for all vibration axes. However, with vibrations in the plane of minimum stiffness, which are considered in this work, the crack on the convex side of the airfoil has an opening mode propagation, as previously stated, which makes it possible to fix its location due to a sharp change in the ratio of the amplitudes of the dominant harmonics along the *Oy* axis.

Conclusions. In this work, the authors show the ratio of harmonics at excitation frequencies that are multiples of the fundamental frequency can be used to diagnose the presence of fatigue crack. To locate the crack, it is necessary to excite the opening crack mode. In this case, in the neighbourhood of the crack, there will be a sharp change in the ratio of the amplitudes of the dominant harmonics at the superharmonic resonance of the order 1/2.

This publication was prepared within the framework of the research project “Development of methods for diagnosis and analysis of the failure causes and reduction of vibration stress level of the blades and their systems during operation of gas turbines”, which has received funding from the National academy of sciences of Ukraine under grant agreement No 240/2021-03/03.

References

1. Dimarogonas A.D. Vibration of Cracked Structures - a State of the Art Review. *Engineering Fracture Mechanics*, 1996, Vol. 55 (5), p. 831-857. https://doi.org/10.1016/0013-7944(94)00175-8
2. Morassi A., Vestroni F. *Dynamic Methods for Damage Detection in Structures*. Wien: Springer-Verlag; 2008.
3. Poursaeidia E., Babaeib A., Mohammadi Arhanic M. R. and Arablua M. Effects of Natural Frequencies on the Failure of R1 Compressor Blades. *Engineering Failure Analysis*, 2012, Vol. 25, p. 304-315. https://doi.org/10.1016/j.engfailanal.2012.05.013
4. Kruts V., Zinkovskii A., Savchenko K. and Onyshchenko Y. Finite Element Analysis of the Open Crack Influence on the Compressor Blade Natural Frequencies, in: *Proc. of the 26th Int. Congress on Sound and Vibration (ICSV 26) (July 7–11, 2019, Montreal, Canada)*, Vol. 4, Curran Associates, Inc. (2019), p. 2305-2310.
5. Shen M.-H.H. and Chu Y.C. Vibrations of Beams with a Fatigue Crack. *Computers & Structures*, 1992, Vol. 45 (1), p. 79-93. https://doi.org/10.1016/0045-7949(92)90347-3
6. Matveev V.V. and Bovsunovskii A.P. Some Aspects of Vibration of an Elastic Body with a "Breathing" Discontinuity of Material. *Strength of Materials*, 2000, Vol. 32 (5), p. 434-445. https://doi.org/10.1023/A:1005202231465
7. Chondros T., Dimarogonas A. and Yao J. Vibration of a Beam with a Breathing Crack. *Journal of Sound and Vibration*, 2001, Vol. 239 (1), p. 57-67. https://doi.org/10.1006/jsvi.2000.3156
8. Matveev V. V., Yakovlev A. P., and Boginich O. E., The influence of amplitude-dependent damping on vibrodiagnostic parameters of the presence of a closing crack in a beam under subharmonic resonance. *Strength of Materials*, 2015, Vol. 47 (3), p. 375-384. https://doi.org/10.1007/s11223-015-9668-5

9. Ebrahimi A., Heydari M. and Behzad M. Forced Vibration Analysis of Rotors with an Open Edge Crack Based on a Continuous Vibration Theory. *Archive of Applied Mechanics*, 2017, Vol. 87, p. 1871–1889. https://doi.org/10.1007/s00419-017-1295-0
10. Savchenko K., Zinkovskii A., Rzadkowski R., Przysowa R. and Kruts V. An influence of shroud design parameters on the static stresses of blade assemblies. *MATEC Web of Conferences*, 304, 03002 (2019). https://doi.org/10.1051/matecconf/201930403002
11. Sinenko E.A. and Zinkovskii A.P. Influence of the Exciting Force Application Point on the Amplitude Spectrum of Flexural Vibrations in a Beam with a “Breathing” Crack. *Strength of Materials*, 2015, Vol. 47 (4), p. 553-560. https://doi.org/10.1007/s11223-015-9689-0
12. Savchenko K., Zinkovskii A., Onyshchenko Ye., Kruts V. and Kabannyk S. On the Vibration of Compressor Blades with a Breathing Crack, in: *Proc. of the 27th Int. Congress on Sound and Vibration (ICSV 27) (July 11–16, 2021, Virtual, Online)*, Silesian University Press (2021), p. 1-6.
13. Zienkiewicz O. C. *Finite Element Method in Engineering Science*. McGraw-Hill Inc; 1971.

Received 05.12.2021

About the Authors / Відомості про авторів / Сведения об авторах

Savchenko Kyrylo Valentynovych (Савченко Кирило Валентинович, Савченко Кирилл Валентинович) – Candidate of Technical Sciences, Senior Researcher of Oscillations and Vibration Reliability Department, G.S. Pisarenko IPS NAS of Ukraine, Kyiv; e-mail: savchenko@ipp.kiev.ua. ORCID: 0000-0002-9690-9758.

Zinkovskyi Anatolii Pavlovych (Зіньковський Анатолій Павлович, Зиньковский Анатолий Павлович) – Corresponding Member of NAS of Ukraine, Doctor of Technical Sciences, Professor, Head of Oscillations and Vibration Reliability Department, G.S. Pisarenko IPS NAS of Ukraine, Kyiv; e-mail: zinkovskii@ipp.kiev.ua. ORCID: 0000-0003-0803-7054.

Onyshchenko Yevheniia Oleksandrivna (Онищенко Євгенія Олександрівна, Онищенко Евгения Александровна) – Candidate of Technical Sciences, Senior Researcher of Oscillations and Vibration Reliability Department, G.S. Pisarenko IPS NAS of Ukraine, Kyiv; e-mail: onyshchenko@ipp.kiev.ua. ORCID: 0000-0001-7550-8544.

Kruts Vadym Oleksiiovych (Круц Вадим Олексійович, Круц Вадим Алексеевич) – Candidate of Technical Sciences, Senior Researcher of Oscillations and Vibration Reliability Department, G.S. Pisarenko IPS NAS of Ukraine, Kyiv; e-mail: kruts@ipp.kiev.ua. ORCID: 0000-0002-9066-047X.

Kabannyk Serhii Mykolaiovych (Кабанник Сергій Миколайович, Кабанник Сергей Николаевич) – Candidate of Technical Sciences, Senior Researcher of Oscillations and Vibration Reliability Department, G.S. Pisarenko IPS NAS of Ukraine, Kyiv; e-mail: kabannyk@ipp.kiev.ua. ORCID: 0000-0002-6315-1987.