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LINEAR VIBRATIONS OF CYLINDRICAL CANTILEVER SHELLS WITHOUT IMPERFECTIONS

Для розрахунку власних частот і форм коливань консольних циліндрових оболонок застосовується метод Релея-Ритца. Застосовуються теорії Сандерса- Коїтера і Доннела. Коливання конструкції розкладаються по ортогональних поліномах. Досліджуються властивості зв'язаних мод коливань. Результати аналізу порівнюються зі скінченно-елементними розрахунками.
Ключові слова: вібрація, консольні циліндрові оболонки, метод Релея-Ритца, ортогональні поліноми.

Для расчета собственных частот и форм колебаний консольных цилиндрических оболочек применяется метод Релея-Ритца. Применяются теории Сандерса-Коитера и Доннела. Колебания конструкции раскладываются по ортогональным полиномам. Исследуются свойства сопряженных мод колебаний. Результаты анализа сравниваются с конечно-элементными расчетами.
Ключевые слова: вибрация, консольные цилиндрические оболочки, метод Релея-Ритца, ортогональные полиномы.

Cylindrical shells are commonly used as elements of rockets, aircrafts and others structures. The natural frequencies and eigenmodes of the linear cantilever cylindrical shells are very important to predict the dynamic behavior of complex engineering structures. The Rayleigh- Ritz method is applied to analyze the eigenfrequencies and the eigenmodes of the cantilever cylindrical shells. The Donnell's and Sanders-Koiter shell theories with orthogonal polynomials are used to study the shell linear vibrations. The eigenfrequencies and the eigenmodes of the cantilever shell are investigated. The eigenfrequencies, which are obtained by these two theories, are close. The obtained results are compared with the data, obtained by software ANSYS. The properties of the conjugate eigenmodes are analyzed. The results of the analysis are compared with the data of finite element calculations.
Keywords: vibration, cantilever cylindrical shells, Rayleigh- Ritz method, orthogonal polynomials.

1. Introduction

Cylindrical shells are commonly used as elements of rockets, aircrafts and others structures. The natural frequencies and eigenmodes of the linear cantilever cylindrical shells are very important to predict the dynamic behavior of complex engineering structures. The most popular shell theories Donnell's, Sanders-Koiter, Flugge-Lur'r-Byrne and Novozhilov are treated in [1-4]. Chiba studied linear and nonlinear vibration of cylindrical clamped-free shells experimentally and theoretically [5-7]. Kurylov, and Amabili[8] studied large -amplitude nonlinear vibration of clamped –free circular cylindrical shell by using Sanders-Koiter nonlinear shell theory. Pellicano [9] used the idea of orthogonal polynomials instead of trigonometric functions to expand the cylindrical shell displacement fields. Kurylov, and Amabili [10] followed this idea and studied nonlinear flexural vibrations of clamped and simply supported cylindrical shells. They used the harmonic functions to approximate the displacements in the circumferential direction and the Chebyshev polynomials for approximation in longitudinal direction.

The linear vibrations of the cylindrical cantilever shell are analyzed by the Rayleigh- Ritz method. The Donnell's and Sanders-Koiter shell theories are used to study the shell linear vibrations. The eigenfrequencies and the eigenmodes of the cantilever shell are investigated. The obtained results are compared with the data, obtained by software ANSYS.

2. The main equations

Thin, clamped-free cylindrical shell is considered (fig. 1). As the shell is thin, the shearand rotary inertia are not taken into account. It is assumed, that the strains and displacements are small. Therefore, the strain-displacements relations are linear. The cylindrical shell without imperfections is analyzed. The elements of the stress tensor and the strain tensor satisfy the Hooke law. Thus, the cylindrical shell performs linear vibrations. The position of the point on the shell middle surface is described by two coordinates (x,θ).

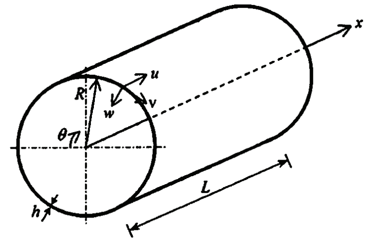

Figure 1 – Circular cylindrical shell

The cylindrical shell dynamics is described by three projections of the displacements $u(x,\theta,t)$; $v(x,\theta,t)$; $w(x,\theta,t)$ on the axes x, θ, t, respectively. The cylindrical shell potential energy takes the following form [11]:

$$\begin{aligned}\Pi=&\frac{1}{2}\frac{Eh}{(1-\nu^2)}\int_0^{2\pi}\int_0^L(\varepsilon_{x,0}^2+\varepsilon_{\theta,0}^2+2\nu\varepsilon_{x,0}\varepsilon_{\theta,0}+\frac{1-\nu}{2}\gamma_{x\theta,0}^2)dxRd\theta+\\&+\frac{1}{2}\frac{Eh^3}{(1-\nu^2)}\int_0^{2\pi}\int_0^L(k_{x,0}^2+k_{\theta,0}^2+2\nu k_{x,0}k_{\theta,0}+\frac{1-\nu}{2}k_{x\theta,0}^2)dxRd\theta+\\&+\frac{Eh^3}{12R(1-\nu^2)}\int_0^{2\pi}\int_0^L\Big(\varepsilon_{x,0}k_{x,0}+\varepsilon_{\theta,0}k_{\theta,0}+\nu\varepsilon_{x,0}k_{\theta,0}+\nu\varepsilon_{\theta,0}k_{x,0}+\\&+\frac{1-\nu}{2}\gamma_{x\theta,0}k_{x\theta,0}\Big)dxRd\theta+O(h^4),\end{aligned} \quad (1)$$

where E is Young 's modulus of the shell material; ν is the Poisson ratio; R is shell radius; h is the shell thickness; L is length of the shell; $\varepsilon_{x,0}$, $\varepsilon_{\theta,0}$, $\gamma_{x\theta,0}$ are elements of the strain tensor; $k_{x,0}$, $k_{\theta,0}$, $k_{x\theta,0}$ are curvatures of the middle surfaces. The first term of potential energy describes stretching and compression of the shell middle surface.

The second and the third terms describe the shell bending.

The Donnell's shell theory is expressed by the following relations between the strains and displacements:

$$\varepsilon_{x,0}=\frac{\partial u}{\partial x};\quad \varepsilon_{\theta,0}=\frac{\partial v}{R\partial x}+\frac{w}{R};\quad \gamma_{x\theta,0}=\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x};$$
$$k_{x,0}=-\frac{\partial^2 w}{\partial x^2};\quad k_{\theta,0}=-\frac{\partial^2 w}{R^2\partial\theta^2};\quad k_{x\theta,0}=-2\frac{\partial^2 w}{R\partial x\partial\theta}. \quad (2)$$

If the Sanders-Koiter shell theory is used, then the equations for the strains are the same as in the Donnell's theory. But the equations for the middle surface curvatures are changed as:

$$k_{x,0}=-\frac{\partial^2 w}{\partial x^2};\quad k_{\theta,0}=\frac{\partial v}{R^2\partial\theta}-\frac{\partial^2 w}{R^2\partial\theta^2};$$
$$k_{x\theta,0}=-2\frac{\partial^2 w}{R\partial x\partial\theta}+\frac{1}{2R}\left(3\frac{\partial v}{\partial x}-\frac{\partial u}{R\partial\theta}\right). \quad (3)$$

The kinetic energy of the cylindrical shell takes the form:

$$T_s=\frac{1}{2}\rho_s h\int_0^{2\pi}\int_0^{L}(\dot w^2+\dot u^2+\dot v^2)\,dxRd\theta\,, \quad (4)$$

where ρ_s is material density.

The cantilever shell is clamped at the edge $x = 0$ and it is free at $x = L$. The following geometrical boundary conditions are satisfied on the clamped edge:

$$u=v=w=\frac{\partial w}{\partial x}=0 \ \text{ at } x=0.$$

The natural boundary conditions are met at the free edge. As the method Rayleigh –Ritz is used to calculate the structure linear vibrations, only the geometrical boundary conditions are accounted [12].

The shell linear vibrations take the following form:

$$W(x,\theta,t)=\widetilde W(x,\theta)\sin\omega t;$$
$$U(x,\theta,t)=\widetilde U(x,\theta)\sin\omega t; \quad (5)$$
$$V(x,\theta,t)=\widetilde V(x,\theta)\cos\omega t.$$

The equations (5) are substituted into (1, 4). Then the kinetic and potential energies can be presented as:

$$T(x,\theta,t)=\omega^2\sin^2(\omega t)\,\overline T\,(x,\theta);$$
$$P\,(x,\theta,t)=\sin^2(\omega t)\overline P\,(x,\theta)\,.$$

The functions $\widetilde W(x,\theta)$; $\widetilde U(x,\theta)$; $\widetilde V(x,\theta)$ can be presented in the form of the double Fourier series as:

$$W(x,\theta)=\sum_{m=1}^{N_1}\sum_{n=1}^{N_2}W_{m,n}\,\phi_m(x)\,\cos(n\theta);$$
$$U(x,\theta)=\sum_{m=1}^{N_3}\sum_{n=1}^{N_4}U_{m,n}\,\chi_m(x)\,\cos(n\theta); \quad (6)$$
$$V(x,\theta)=\sum_{m=1}^{N_5}\sum_{n=1}^{N_6}V_{m,n}\,\chi_m(x)\sin(n\theta)\,,$$

where ϕ_m, $\chi_m(x)$ are trial functions; $W_{m,n},U_{m,n},V_{m,n}$ are unknown coefficients, which are determined by the Rayleigh- Ritz method.

Several types of approximations are used for the displacements (6). In the first case, it is considered the beam functions for $\phi_m(x)$ and orthogonal polynomial for $\chi_m(x)$. In the seond case, it is used the orthogonal polynomial for both $\phi_m(x)$ and $\chi_m(x)$. It is obtained these orthogonal polynomials. The following set of polynomials $\{x, x^2, x^3, \ldots, x^n\}$ is used. The Gram-Schmidt orthogonalization is applied for these polynomials. As a result, the sequence of the orthogonal polynomials $\{\chi_1, \chi_2, \chi_3, \ldots, \chi_m\}$ is obtained. These polynomials are used for the functions $\phi_1(x),\phi_2(x),\ldots$ too.

The shell linear vibrations are satisfied the minimum of the following functional [12]:

$$\int_0^{2\pi/\omega}(P-\mathrm{T})dt=\frac{\pi}{\omega}\left[\overline P\left(W_{1,1},\ldots,V_{N_5,N_6}\right)-\omega^2\overline{\mathrm{T}}\left(W_{1,1},\ldots,V_{N_5,N_6}\right)\right]. \quad (7)$$

Minimum of the functional (7) is obtained on the set of $X=\left\{W_{1,1},\ldots,V_{N_5,N_6}\right\}$. The conditions of the minimum of the functional have the following form:

$$\frac{\partial}{\partial W_{mn}}(\overline P-\omega^2\overline T)=0,(n=1\ldots N_1,m=1..N_2)\,;$$
$$\frac{\partial}{\partial U_{mn}}(\overline P-\omega^2\overline T)=0,(n=1\ldots N_3,m=1..N_4)\,; \quad (8)$$
$$\frac{\partial}{\partial V_{mn}}(\overline P-\omega^2\overline T)=0,(n=1\ldots N_5,m=1..N_6)\,.$$

The set of the equations (8) are transformed into the following eigenvalue problem:

$$Det\left[C-\omega^2 M\right]=0, \quad (9)$$

where C, M are stiffness and mass matrixes.

Thus, the parameters of the cantilever shell linear vibrations are obtained from the eigenvalue problem (9).

3. Analysis of linear vibrations

In this section the shell linear vibrations are analyzed numerically. The numerical results are compared with the data obtained by ANSYS. The calculations are carried out for the shell with the following numerical values of parameters:

$$L = 0.48\text{ m};\ h = 0.178\cdot10^{-3}\text{ m};\ E = 6.82\cdot10^{10}\text{ Pa};$$
$$\rho = 27122\text{ kg/m}^3;\ \nu = 0.3;\ R = 0.074\text{ m}. \quad (10)$$

The eigenvalue problem (9) is solved to calculate eigenfrequencies and eigenmodes of the shell. The obtained results are compared with the results, published by Kurilov and Amabili [5] and by Leissa [4].

The first case of the displacements approximation is considered. Then the beam functions are used for $\phi_m(x)$ and the orthogonal polynomials are used for $\chi_m(x)$. The results of the analysis are published in Table 1. The data, obtained by the software ANSYS, is published in the second row of the Table. The data, which are published in the papers [5] and [4], are shown on the third and fourth rows of the Table. The results, obtained by the Rayleigh-Ritz method with Donnell's theory (2), are published in the fifth row of the Table. The results, obtained by the Rayleigh- Ritz method with Sanders-Koiter's shell theory (2, 3), are published in the sixth row of the Table.

Let us consider the case, when the orthogonal polynomials are used for both $\phi_m(x)$ and $\chi_m(x)$ functions of the expansion (6). The data of the calculations by the Donnell theory and the Sanders-Koiter theory are published in the seventh and the eight columns of the Table, respectively.

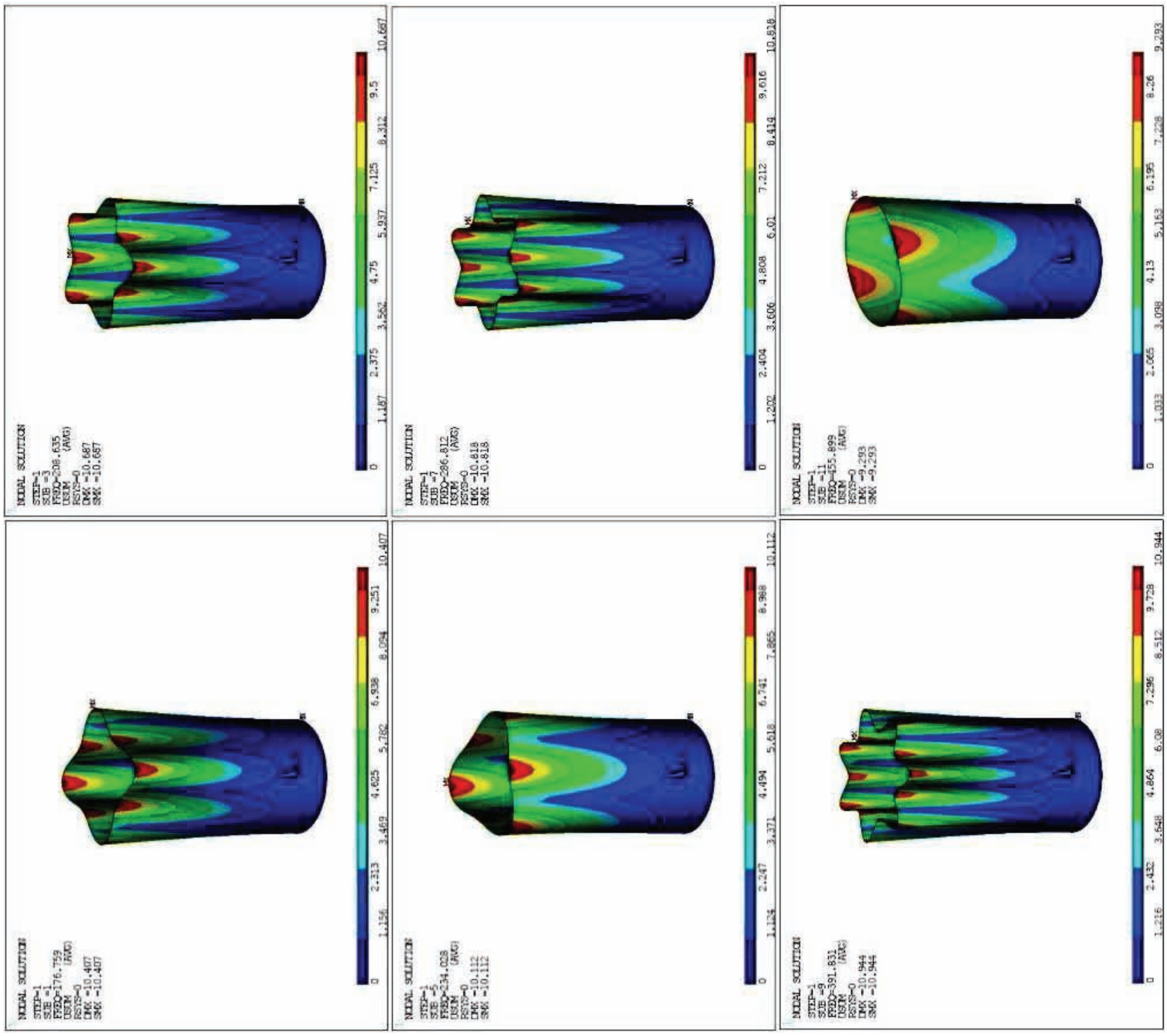

Figure 2 – Eigenmodes of the shell vibrations

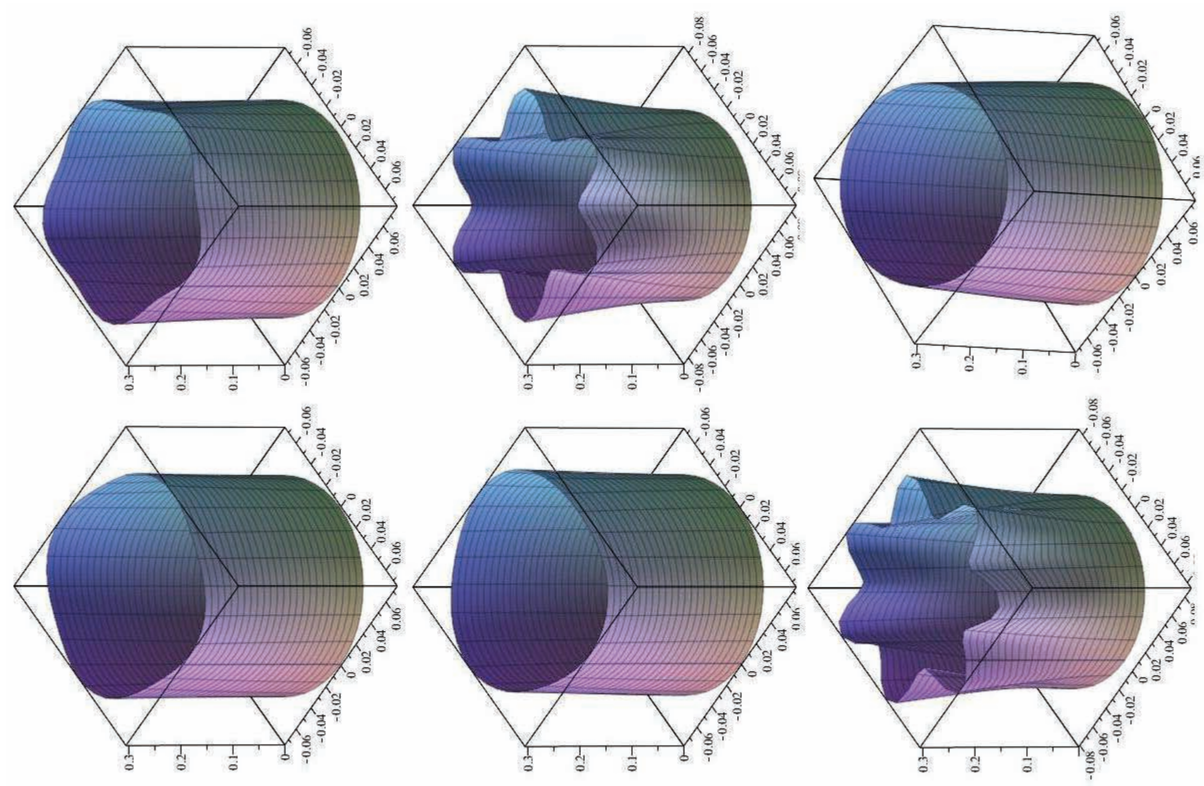

Figure 3 – Eigenmodes of the shell vibrations, which is obtained by the Rayleigh- Ritz method

Table 1 – The eigenfrequencies of the cantilever shell

Number of eigenfrequency	1	2	3	4	5	6	7	8
Ansys	176.76	208.63	234.03	286.81	391.83	455.9	465.92	484.09
Kurylov	175.5	205.3	233.9	377.6	456.4	494.1	627	776
Leissa	181	207	246	378	489	494	456.029	527.859
Donnell	181.694	212.78	287.366	385.798	463.291	466.885	478.982	502.412
Sanders-Koiter	176.645	205.801	236.358	279.814	378.814	462.183	472.696	494.636
Donnell1	181.1	212.6	236.4	287.3	385.8	457.5	464.8	478.2
Sanders-Koiter1	176.1	205.7	234.4	279.8	378.1	457.1	460.1	471.9

Fig. 2 shows the eigenmodes of the shell bending vibrations obtained by software ANSYS and Fig. 3 shows the eigenmodes of the shell obtained by Rayleigh- Ritz method. The shell bending vibrations in the shell longitudinal direction are governed by the eigenmode of the cantilever beam. Note, that the first seven eigenmodes of the bending vibrations are governed by the first eigenmode of the cantilever beam. The first and the second eigenmodes of the bending vibrations contain eight and ten nodes in the circumference direction, respectively. The node number of the eigenmodescan be followed from Fig.2.

Conclusion

The linear vibrations of the cylindrical cantilever shell are analyzed by the Rayleigh- Ritz method. The Donnell's and Sanders-Koiter shell theories are used to study the shell linear vibrations. The eigenfrequencies, which are obtained by these two theories, are close. The obtained results are compared with the data, obtained by software ANSYS.

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