RESEARCH OF NONLINEAR VIBRATIONS OF LAMINATED SHALLOW SHELLS WITH CUTOUTS BY R-FUNCTIONS METHOD

In present work an effective method to research geometrically nonlinear free vibrations of elements of thin-walled constructions that can be modeled as laminated shallow shells with complex planform is applied. The proposed method is based on joint use of R–functions theory, variational methods and Bubnov–Galerkin procedure. It allows reducing an initial nonlinear system of motion equations of a shallow shell to the system of nonlinear differential equations of motion [5, 6]:

\[
\frac{\partial^2 N_{11}}{\partial x^2} + \frac{\partial^2 N_{12}}{\partial x \partial y} = m_1 \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial^2 N_{22}}{\partial y^2} + \frac{\partial^2 N_{12}}{\partial x \partial y} = m_2 \frac{\partial^2 v}{\partial t^2},
\]

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + k_1 N_{11} + k_2 N_{22} + N_{11} \frac{\partial^2 w}{\partial x^2} + 2 N_{12} \frac{\partial^2 w}{\partial x \partial y} + N_{22} \frac{\partial^2 w}{\partial y^2} = m_1 \frac{\partial^2 \psi}{\partial t^2};
\]

\[
\frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial y} + Q_x = m_1 \frac{\partial^2 \psi}{\partial t^2},
\]

\[
\frac{\partial M_{22}}{\partial y} + \frac{\partial M_{12}}{\partial x} + Q_y = m_1 \frac{\partial^2 \psi}{\partial t^2},
\]

where \(u(x,y,t), v(x,y,t), w(x,y,t)\) are displacements of the coordinate surface points; \(\psi, \phi\) are rotation angles of the normal to the coordinate surface; \(N_{11}, N_{12}, N_{22}\) are the in-plane resultants per unit length; \(M_{11}, M_{12}, M_{22}\) are the internal moment resultants per unit length; \(Q_x, Q_y\) are the transverse shear resultants per unit length. Components of these resultants are defined as:

The mathematical statement of the problem

We consider the multi-layered thin shell of the constant thickness \(h\), on the assumption that the slip and separation between the layers are absent. We confine ourselves to the symmetrical structure of layers. The mathematical formulation of the problem is performed via refined theory of multi-layered shells based on the Timoshenko’s shear assumptions. Then the problem of geometrically nonlinear vibrations of shallow shells is reduced to the solution of the system of nonlinear differential equations of motion [5, 6]:

\[
\frac{\partial N_{11}}{\partial x} + \frac{\partial N_{12}}{\partial y} = m_1 \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial N_{22}}{\partial y} + \frac{\partial N_{12}}{\partial x} = m_2 \frac{\partial^2 v}{\partial t^2},
\]

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + k_1 N_{11} + k_2 N_{22} + N_{11} \frac{\partial^2 w}{\partial x^2} + 2 N_{12} \frac{\partial^2 w}{\partial x \partial y} + N_{22} \frac{\partial^2 w}{\partial y^2} = m_1 \frac{\partial^2 \psi}{\partial t^2};
\]

\[
\frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial y} + Q_x = m_1 \frac{\partial^2 \psi}{\partial t^2},
\]

\[
\frac{\partial M_{22}}{\partial y} + \frac{\partial M_{12}}{\partial x} + Q_y = m_1 \frac{\partial^2 \psi}{\partial t^2},
\]

where \(u(x,y,t), v(x,y,t), w(x,y,t)\) are displacements of the coordinate surface points; \(\psi, \phi\) are rotation angles of the normal to the coordinate surface; \(N_{11}, N_{12}, N_{22}\) are the in-plane resultants per unit length; \(M_{11}, M_{12}, M_{22}\) are the internal moment resultants per unit length; \(Q_x, Q_y\) are the transverse shear resultants per unit length. Components of these resultants are defined as:
where \( \psi \) are the components of \( \psi \),

\[
N_x = \begin{bmatrix} C_{11}C_{12}C_{16}K_{11}K_{12}K_{16} \\ C_{12}C_{22}C_{26}K_{12}K_{22}K_{26} \\ C_{16}C_{26}C_{66}K_{16}K_{26}K_{66} \end{bmatrix}, \quad N_y = \begin{bmatrix} K_{11}K_{12}D_1D_1D_6 \\ K_{12}K_{22}D_1D_2D_2D_6 \\ K_{16}K_{26}D_6D_6D_6 \end{bmatrix}, \quad N_{xy} = \begin{bmatrix} \psi_{x,x} \\ \psi_{x,y} \\ \psi_{y,y} \end{bmatrix}, \quad M_x = \begin{bmatrix} K_{11}K_{12}D_1D_1D_1D_1D_6 \\ K_{12}K_{22}D_1D_2D_2D_2D_6 \\ K_{16}K_{26}D_6D_6D_6D_6 \end{bmatrix}, \quad M_{xy} = \begin{bmatrix} \psi_{x,x} + \psi_{y,y} \end{bmatrix},
\]

\[
Q_x = k_1^2C_{11} \left( \frac{\partial^2 w}{\partial x^2} + \psi_x \right) + k_2^2C_{45} \left( \frac{\partial^2 w}{\partial y^2} + \psi_y \right),
\]

\[
Q_y = k_1^2C_{44} \left( \frac{\partial^2 w}{\partial x^2} + \psi_x \right) + k_2^2C_{45} \left( \frac{\partial^2 w}{\partial y^2} + \psi_y \right).
\]

The expressions of deformations, taking nonlinear terms in account, are expressed as

\[
\varepsilon_x = \frac{\partial w}{\partial x} - k_1w + \frac{1}{2} \left( \frac{\partial^2 w}{\partial x^2} \right)^2; \quad \varepsilon_y = \frac{\partial w}{\partial y} - k_1w + \frac{1}{2} \left( \frac{\partial^2 w}{\partial y^2} \right)^2;
\]

\[
\varepsilon_{xy} = \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w \partial^2 w}{\partial x \partial y}.
\]

The expressions (2) contain coefficients \( C_{ij}, K_{ij}, D_{ij} \) and \( m_1, m_2 \), which are calculated by the known formulas [7-10], \( k_1, k_2 \) are shear correction coefficients, \( k_1, k_2 \) are curvatures of a shell.

The system (1) is supplemented by appropriate boundary and initial conditions.

1.1 The solution method

Accordingly to the algorithm proposed in [8-10], we represent the unknown functions \( u(x,y,t), v(x,y,t), w(x,y,t), \psi_x(x,y,t), \psi_y(x,y,t) \) in the form of an expansion by eigenfunctions of the corresponding linear problem solution of free vibrations of shells

\[
w(x,y,t) = y_1(t) \cdot w_1(x,y); \quad \psi_x(x,y,t) = y_1(t) \cdot \psi_{x_1}(x,y);
\]

\[
\psi_y(x,y,t) = y_1(t) \cdot \psi_{y_1}(x,y);
\]

\[
u(x,y,t) = y_1(t) \cdot v_1(x,y) + y_1(t) \cdot v_2(x,y);
\]

\[
\psi_x(x,y,t) = y_1(t) \cdot \psi_{x_1}(x,y) + y_1(t) \cdot \psi_{x_2}(x,y).
\]

The functions \( u_1, v_1, \psi_{x_1}, \psi_{y_1} \) are the components of the eigenvector \( \hat{U} = (u,v,w,\psi_x,\psi_y) \), and functions \( u_2, v_2 \) must be a solution of the system of differential equations

\[
\begin{cases}
L_1u_2 + L_{12}v_2 = -N_1^{(2)}(w_1) \\
L_{12}u_2 + L_2v_2 = -N_2^{(2)}(w_1)
\end{cases}
\]

The expressions for the right-hand sides of (4), designated by operators \( N_k^{(2)}(w_1), k = 1,2 \) are described in [8-10].

The system of equations (4), supplemented with the appropriate boundary conditions, as well as a linear problem of free oscillations of multi-layered shallow shells, can be solved using RFM method [3, 4] for almost any form of the shell’s plan and various types of boundary conditions.

Substituting the expression (3) for the unknown functions in the system (1) and applying the Bubnov-Galerkin procedure, we obtain the nonlinear ordinary differential equation

\[
y_1^3(t) + \alpha y_1(t) + \beta \cdot y_1^3(t) + \gamma \cdot y_1(t) = 0.
\]

The observed divergence of obtained results with the once of [11] does not exceed 2 %.

Example 2. Consider the problem of a 3-layered cylindrical shallow shell free nonlinear vibration with a square planform and a central square cutout (fig. 2).

\[\text{Figure 1 – Convergence of amplitude-frequency curves of shallow shells}\]

\[\text{Figure 2 – Convergence of amplitude-frequency curves of shallow shells}\]

\[\text{Figure 3 – Convergence of amplitude-frequency curves of shallow shells}\]
It is assumed that all layers are made of the material with the following characteristics:

Material 1 (M1): \( E_1 = 40E_2; G_{12} = G_{13} = G_{23} = 0.5E_2; \)
\( \nu_{12} = 0.25. \)

Material 2 (M2): \( E_1 = 10E_2; G_{12} = G_{13} = G_{23} = 0.5E_2; \)
\( \nu_{12} = 0.25. \)

The shear correction factors are taken as \( k_1 = k_2 = 5/6, \) dimensionless parameters of the curvature are defined as \( R_i/h = 300; h/(2a) = 0.01. \) Three types of boundary conditions are investigated: completely clamped edge (CC), simply supported on external edge and clamped cutout (SC), simply supported on external edge and free cutout (SF). Comparison of the obtained fundamental frequencies for two types of materials (M1, M2) with the once in [7] is presented in table 1. Results presented by RFM were obtained both by polynomial approximation (POLY) and splines [12] (SPLI) using mesh of 10x10.

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The observed divergence of obtained results with the once of [7] does not exceed 3%.

Further, new results are presented by using the theory of R–functions.

The amplitude-frequency dependence for cylindrical shells of SC boundary condition with the cutout of size \( c/a = 0.2 \) for two types of materials (M1, M2) is presented in figure 3.

Figure 3 – Amplitude-frequency curves of SC cylindrical shallow shells

According to the observed curves we can state that the behavior of investigated shell of M1 material is more rigid than the one of M2 with the amplitude increase.

The amplitude-frequency dependence for cylindrical shells of CC boundary condition with the cutout of size \( c/a = 0.2 \) for two types of materials (M1, M2) is presented in figure 4.

According to the observed curves we can state that the investigated shell of M1 material becomes more rigid than the one of M2 when the ratio \( W_{max}/h \) exceeds 1.4.

Figure 4 – Amplitude-frequency curves of CC cylindrical shallow shells

Conclusions

A proposed numerical-analytically approach based on R–functions theory is used to research free nonlinear vibration problems of laminated shallow shells with cutouts. Three-layered shells made of different materials with different curvatures and square cutout are investigated. Different types of boundary conditions are examined. The amplitude-frequency curves of vibrations of considered
shells have been constructed using the first-mode approximation by the Runge-Kutta method. A comparison with known results confirms the reliability of the proposed approach.

References:

Bibliografічні описи / Bibliographic descriptions


Відомості про авторів / Сведения об авторах / About the Authors

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