This paper studies the problem of whirl vibrations of rotating drill string bit under conditions of its contact interaction with the borehole bottom surface. The destructive effects of whirl vibrations of drill bits and drill strings have long been recognized. The backward whirling is believed to randomly happen to some degree in 40% of all wells.

It is shown that the situation is essentially associated with enlargement of the compressive thrust force and its approaching to the Eulerian critical value. Then, the bending stiffness of the drill string reduces, it hogs and the diamond impregnations existing in the bit surface penetrate into the rock medium. As this takes place, the bit axis tilts and nutates. It should be particularly emphasized that at this stage, the bit is under action of the vertical force (weight on bit) which essentially exceeds all other forces. It presses the bit to the bore-hole wall and then instantaneous center of its motion changes from being in sliding contact to rolling contact [1-3]. But in our opinion, a near-zero transverse component called the bit imbalance force.

In the majority of cases, previous studies of drill bit whirl have concentrated primarily on assumption that in whirling the rotating drill bit is compressed to the bore-hole wall and is rolling on its surface [1-3]. But in our opinion, a prominent phase of this effect proceeds at the initial stage of its generation. These phenomena occur when the drilling bit changes over from being in sliding contact to rolling contact with the bore-hole bottom and then instantaneous center of its rotation begins to move around the DS axis on the bore-hole bottom surface [4]. Depending on the rotative movement direction, the forward and backward whirls are distinguished [2, 5]. As this takes place, the backward whirl can achieve frequencies somewhere in the range from 5 to 30 times the speed of the DS rotation [4]. This speed makes the generated force effect especially detrimental, producing a set of high fatigue-loading and strong impact-loading conditions that can be responsible for intensive bending vibration and irremediable detriment to downhole tools when the bit does not achieve the well wall, does not touch it, and is rolling on the well bottom.

In this case, the most influential axial force can be resolved into a vertical force or weight on bit (WOB) and a near-zero transverse component called the bit imbalance force. This force is usually referred to as a percentage of the WOB. Numerous measurements of the bit imbalanced force evidence that commercial bits might be 2% imbalanced, nevertheless, 10% imbalanced is more typical. Hence, it can be concluded that the bit is compressed to the well bottom by the WOB force, while only 10% of its value are responsible for the tendency to compress it to the well wall. Therefore, it is more reasonable to associate these vibrations with the bit rolling on the bottom surface rather than on the well, as is traditionally considered in theoretical researches.

According to field observation, the main factor influencing whirl is the bit geometry. In papers [3, 5], the phenomena of the bits spinning and rolling on the well bottom are simulated with the use of nonlinear frictional and


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nonholonomic models for bits with different shapes. As an example, spherical bit whirling on spherical bore-hole bottom is numerically studied. It permitted to find three types of its stable and unstable motions associated with forward and backward rolling as well as pure spinning.

The problem on whirling of rigid ellipsoidal bits on well bottoms is considered in [4]. The oblong and oblate ellipsoidal shapes are selected for investigation. Two mathematical models of the bit whirling founded on supposition of the capability of their pure rolling whirl and rolling with sliding are proposed. The unsteady regimes of their rolling initiated by their initial perturbations are analyzed through computer simulations. The oblate bits are shown to be more disturbance-sensitive and curves of their whirling trajectories have anisiform outlines with sharp turns. Summarizing, it may be noted that the effects considered in these papers are simulated on the basis of nonlinear statements. They are multi-parametric as depend on elastic, dynamic, geometric, kinematic, frictional, and structural characteristics of the system. Therefore, the particular cases treated by computer simulations are only intriguing and inviting attention to their beauty. To establish general regularities of these processes proceeding, the object-oriented analytic studies should be performed. They cannot be performed through the application of created non-linear mathematic models and should be oriented to incipient stage of the process evolvement which can be described by linearized equations.

1 Whirling of a spheric bit on spheric bottom surface

To elaborate mathematic model of a drill bit whirling, take into account that this vibration process is attended with elastic bending of the DS tube adjacent to its bottom hole assembly (BHA). Assume that it is possible to separate mentally the lower sections AB and BC located between two lower centralizers A and B, and the collar boundary C with the bit at its end (Fig. 1). In the model constructing, the feature of fundamental importance is simulation of the bit motion which can be dynamic or kinematic. To perform the choice, it is necessary to compare frequencies of free vibrations of the separated piece of the DS and angular velocity of the bit. For example, the first frequency of free vibrations of the steel three-point beam ABC with lengths \( e = 2 \text{ m}, \ l = 9 \text{ m} \) (Fig.1) and cross-section diameters \( d_1 = 0,18 \text{ m}, \ d_2 = 0,18 \text{ m} \) equals \( f_1 = 106,7 \text{ s}^{-1} \), while the DS angular velocity may achieve only \( \omega \approx 10–20 \text{ s}^{-1} \). Besides, the bit represents a hollow-centered body with small inertia moment. So, the considered whirlings represent quasi-static processes of the DS deforming, inertia forces can be neglected at its analysis, and the whirling motions can be reckoned as kinematic, proceeding with velocities determined by angular velocity \( \omega \) of the DS rotation. Therefore, in the whirling study only elastic and constraint forces will be taken into account and kinematic stimulation of the system motion will be analyzed.

2 Model of a spherical bit rolling on a spherical bottom surface

Assume that a DS is elastically deflected from vertical and its spherical bit, rotating with angular velocity \( \omega \) around its axis, begins to roll on uneven spherical surface of the well bottom \( \pi \) (Fig. 2). Then, depending on the position of contact point \( G \) with respect to the \( D \) point, lying in the rotation axis, the bit center \( C \) can rotate in the direction of the bit rotation (forward whirling – Fig. 2, \( a \)) or, every so often, in the opposite direction (backward whirling – Fig. 2, \( b \)). Introduce immoveable coordinate system \( OXYZ \) and reference frame \( Oxz \) fixed in the beam of the rotating DS. Vertical axes \( OZ \) and \( Oz \) coincide. As is traditional in nonholonomic mechanics [5], it is convenient to examine this motion relative to its instantaneous center \( G \) of total rotation, because in this case the unknown constraint reaction \( \mathbf{R} \) is excluded from the constitutive equations and only elastic axial (\( T \)) and shear (\( Q \)) forces and bending moment (\( \mathbf{M} \)) participate in the force equilibrium. Let the radii of the bit and hole bottom surfaces be \( a \) and \( b \), correspondingly.

If to assume that the contacting surfaces are absolutely uneven, then the sliding effects becomes impossible. In this case, \( v = 0 \) and pure rolling with spinning is realized. Such model belongs to the nonholonomic type. It is shown in paper [4] that the possibility of bit sliding the bore-hole bottom is severely reduced through the presence of diamond thorns on its surface. This effects is amplified by enlargement of the bit pressure on the hole bottom and decrease of the DS stiffness. Then, the nonholonomic model application is thoroughly justified.

In paper [3], nonlinear kinematic and dynamic equations were constructed, stemming from conditions of the spherical drill bit rolling without sliding and equilibrium of elastic moment \( \mathbf{M} \) and shear force \( \mathbf{Q} \) applied to the bit.
A special feature of this system of four equations lies in the fact that it contains eight unknown values \( u, u', \alpha, \beta, v, v', \gamma, \gamma' \) prescribed at the C edge of the DS rotation in fixed coordinate system. Value such as \( p = 0 \) in this situation, they testify that in the fixed reference frame, the bit moves with angular velocity \( \Omega_1 = \omega(p - 1) \) and its motion is determined by equations:

\[
X(t) = C_1 \sin \Omega_1 t + C_2 \cos \Omega_1 t; \\
Y(t) = -C_1 \sin \Omega_1 t + C_2 \cos \Omega_1 t.
\]  

Eqs. (8) and Eqs. (10) permit one to analyze the possible tendencies for the spherical bit to perform different self-triggered whirlings depending on the frequency \( p \) values. Thus, if \( p < 1 \), then, as Eqs. (8) and (9) testify, the bit center moves along circular trajectory with angular velocity \( \Omega_1 = \omega(p - 1) \) outstripping the DS rotation in fixed coordinate system \( OXYZ \). Value \( p = 1 \) is in line with \( \Omega_1 = \omega \), \( \Omega_2 = 2\omega \).

Special regime is generated at \( p = 0 \). In this situation, the bit is motionless in rotating reference frame (\( \Omega_1 = 0 \)) but performs forward whirling with frequency \( \Omega_2 = \omega \) in
still space. The opposite situation occurs at \( p = 1 \) when the bit rotates in reverse direction with frequency \( \Omega_2 = \omega \), though comes to a standstill at point \( X = C_2 , Y = C_1 \) in the still space. Accentuate that this state is also unfavorable because here the bit performs pure spinning without rolling and sliding and the drilling direction begins to change its direction.

The subsequent enlargement of the parameter entails change-over from forward whirling of the bit to backward motion with further essential enhancement of the \( \Omega_2 \) frequency. Note once again that this detrimental effect is in compliance with the unexpected field observations cited in paper [3] and consisting in essential increase in the whirl frequency which can be 30 times greater than the angular velocity of the DS, while the generally recognized model of the bit touch with the bore-hole wall [2] cannot reveal it.

The established correspondences between the values of parameter \( p \) and types of the spherical bit whirling allow one to simulate the modes of its motion at the initial stage of its perturbation. But now, the further interest is provoked by the question of how the \( p \) values change with varying of the system parameters. Among the most essential values affecting the whirling stimulation are bending stiffness \( EI \) of the DS, radii of the bit a and well bottom surface \( b \), spans \( l \) and \( e \), WOB force \( T \), and correlations between parameters of elasticity \( w_{m}, w_{q}, w_{m}^{(l)} \) and \( w_{q}^{(l)} \). Assume that typical magnitudes of these values for steel DSs comprise \( EI = 4.07 \times 10^6 \, \text{Pa} \cdot \text{m}^4 ; l = 9 \, \text{m} ; e = 2 \, \text{m} ; a = 0.1 \, \text{m} ; b = 0.3 \, \text{m} ; w_{m} = 1.968 \times 10^{-6} \, \text{m} ; w_{q} = 3.618 \times 10^{-6} \, \text{m} ; w_{m}^{(l)} = 1.23 \times 10^{-6} ; w_{q}^{(l)} = 1.97 \times 10^{-6} ; T = -1 \times 10^4 \, \text{N} \) In this case, \( p = 1,376 \) and the whirling proceeds with the frequencies \( \Omega_1 = 1,376 \omega \) in the rotating reference frame and \( \Omega_2 = 0,376 \omega \) in the fixed coordinate system.

Acknowledgments

The authors gratefully acknowledge the financial support from the Ministry of Education and Science of Ukraine (project 0115 U 002270)

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Надійшла (received) 06.09.2016

Бібліографічні описи / Bibliographic descriptions


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