DANGEROUS BIFURCATIONS IN 2-DOF VIBROIMPACT SYSTEM

Dynamic behaviour of strongly nonlinear non-smooth discontinuous vibroimpact system is studied. Under variation of system parameters we find the discontinuous bifurcations that are the dangerous ones. It is phenomenon unique to non-smooth systems with discontinuous right-hand side. We investigate the 2-DOF vibroimpact system by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods. We simulate the impact by nonlinear contact interactive force according to Hertz’s contact law. We find the discontinuous bifurcations by Floquet multipliers values. At such points set-valued Floquet multipliers cross the unit circle by jump that is their moduli becoming more than unit by jump. We also learn the bifurcation picture change when the impact between system bodies became the soft one due the change of system parameters. This paper is the continuation of the previous works.

Keywords: Vibroimpact, Discontinuous, Hertz’s law, Bifurcation, Multiplier, Nonlinear, Stability.

Introduction. Nonlinear problems are arising in many different domains of science and engineering. Often they are modeled using sets of ordinary differential equations with discontinuous right-hand side. For example they are the systems with mechanical impacts, stick-slip motion from friction, electronic switches, hybrid dynamics in control, and genetic networks [1]. Vibroimpact system is one example of such systems. Vibroimpact system is strongly nonlinear non-smooth one; the set of its motion differential equations contains the discontinuous right-hand side. Many new phenomena unique to non-smooth systems are observed under variation of system parameters. Jumps and switches in a system’s state represent the grossest form of nonlinearity. Recently the investigations of such systems are developed rapidly. But today it has become clear that many aspects of dynamical behaviour of non-smooth systems aren’t investigated and understood. Especially systems with impacts are of the particular interest for scientists. Under variation of system parameters a nonlinear system can often exhibit catastrophic bifurcations that destroy the desirable system state. Discontinuous bifurcations which occur in non-smooth vibroimpact systems are dangerous ones. They are hard bifurcations. Just such hard bifurcations can perturb the crisis and catastrophe [2–4].

A crisis is a sudden discontinuous change in a chaotic attractor as a system parameter is varied. The crisis can be considered as a catastrophe that one endeavours to avoid. Catastrophic events can occur in different form in various kinds of nature, physics and mechanic systems. After the crisis the system state is quite different from that one before the crisis. If the nonlinear dynamical system state before the crisis is normal and desirable then the state after the crisis may be undesirable or destructive. The hard bifurcations were the subject of Catastrophe theory. Catastrophe theory was introduced in the 1960s by the renowned Field Medal mathematician Rene Thom as a part the general theory of local singularities [5]. Since then it has found applications across many areas, including biology, economics, and chemical kinetics. By investigations the phenomena of bifurcation and chaos, Catastrophe theory proved to be fundamental to the understanding of qualitative dynamics. The famous books [6, 7] are devoted to this topic. The theory was very fashionable at 70th years of 20th century. Then this fashion went away and terminology from catastrophe returned to singularities, discontinuous bifurcations and so on. But the catastrophes and crises remained. Blue Sky Catastrophes, the Swallow’s Tail bifurcations are learnt by contemporary scientists [8].

The bifurcation analysis execution and the bifurcation diagrams building allow to find and to distinguish the safe, explosive, and dangerous bifurcations in dissipative dynamical systems. There are crying needs for investigations of arising of the safe, explosive and dangerous bifurcations in dynamical systems, of the crises and catastrophes for chaotic attractors. We have observed the fold catastrophe in two-body 2-DOF vibroimpact system [9–11].

We investigate the dynamic behaviour of 2-DOF vibroimpact system by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods. Short review was made in [10]. The works [12, 13] were discussed in this survey. We simulate the impact by nonlinear contact interactive force according to Hertz’s contact law. Such simulation gives us the possibility to find the motion law along the whole time-base including the impact phase, to determine the impact duration and to find the contact impact forces. We find discontinuous bifurcation points where set-valued Floquet multipliers cross the unit circle by jumps that is their moduli becoming more than unit by jump. It is phenomenon unique for nonsmooth systems with discontinuous right-hand side. We also learn the change of vibroimpact system dynamical behaviour when the impact between system bodies became the soft one due the change of system parameters. This paper is the continuation of the previous works [9–11].

The aims of this paper are:
1. To find discontinuous bifurcations which can be the dangerous ones under variation of excitation amplitude for strongly nonlinear 2-DOF vibroimpact system.
2. To find discontinuous bifurcations under variation of excitation frequency.
3. To analyze the change of vibroimpact system bifurcation behaviour under the impact softening.

Problem formulation. The initial equations. So far as this paper is the continuation of works [9–11] the prob-
This vibroimpact system is formed by the main body and attached one, and the latter can play the role of percussive or non-percussive dynamic damper. Bodies are connected by linear elastic springs and dampers. The main body is under the effect of periodical external force:

$$F(t) = F\cos(\omega t + \varphi) \quad (1)$$

We consider impacts as low velocity elastic collinear collisions without friction. The contact surfaces are smooth curvilinear ones without roughness. Thus real surface geometry in contact zone may be approximated by «Herzian» geometry.

The initial point of x coordinate is chosen in the main body mass center at the moment when all springs are not deformed. The initial distance between bodies at this moment is D. The structure of the system is experiencing transformation during the movement. The reason is its moment is D. The system is experiencing local deformation in contact zone, $H(x_1 - x_2)$ is the Heaviside step function, $v_1$ and $E_i$ are respectively Poisson’s ratios and Young’s modulus for both bodies, $A$, $B$ and $q$ are the geometry characteristics of contact zone. We consider these surfaces as spherical ones, then $A = B = 1/2R_1 + 1/2R_2$, where $R_1$, $R_2$ are the contact surfaces radiuses. Only local deformations in contact zone are taken into account by the Hertz’s theory. There are different proposals to make Hertz’s formula more precise. Nevertheless Hertz’s theory is widely used for analysis of vibroimpact system dynamics now too. Just impact simulation by nonlinear contact interaction force allows to find the motion law at all timebase including impact phase, to define impact duration and contact forces values.

**Bifurcation analysis.** Bifurcation analysis of vibroimpact system dynamic behaviour was fulfilled by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods [9]. Periodic motion stability or instability was determined by matrix monodromy eigenvalues that is by Floquet multipliers’ values. The periodical solution is becoming unstable one if even though one Floquet multiplier leaves the unit circle in complex plane that is its modulus becoming more than unit. Such multiplier value characterizes the bifurcation kind of this bifurcation point. We have described the theoretical basis for analysis of two-body 2-DOF system in [9], numerical system parameters are given in [9–11].

**Discontinuous bifurcations under excitation amplitude varying.** We have plotted out the oscillation amplitude dependence on excitation amplitude for both system bodies that is the loading curves. Their global view in wide range of excitation amplitude is given at [10]. Now we’ll look at their partial view where discontinuous bifurcation occurs (Fig. 2). Here and further the upper curve corresponds to attached body, the lower one – to main body. Unstable regimes are dotted by red colour. The oscillation amplitude is calculated as

$$A_{\text{max}} = (x_{\text{max}} + |x_{\text{min}}|)/2.$$
whose equations have discontinuous right-hand side. The vibroimpact system converts its motion from impactless one (section OB) into motion with periodic impacts – unstable (1,1)-regime. Other regimes – stable and unstable branches of (5,2)-regime\(^1\) – are arising here. At point B Floquet multipliers are experiencing a discontinuous change and accepting big values [18, 19].

The set-valued Floquet multipliers cross the unit circle in direction of real positive axis by jump that is their moduli becoming more than unit by jump. Floquet multipliers behaviour in the excitation amplitude range \(0 < P < 500 \text{ N}\) is shown at Fig. 3. Fig. 4 also shows well these jumps for multipliers \(\mu_1\) and \(\mu_2\).

Table 1 shows these jumps by numbers.

Naturally the contact force also has a discontinuous bifurcation at point B.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\(P\) & Re(\(\mu_1\)) & Im(\(\mu_1\)) & |\(\mu_1\)| & Re(\(\mu_2\)) & Im(\(\mu_2\)) & |\(\mu_2\)| \\
\hline
98.48 & -0.25 & 0.83 & 0.87 & -0.25 & 0.83 & 0.87 \\
98.98 & -0.25 & 0.83 & 0.87 & -0.25 & 0.83 & 0.87 \\
99.48 & 3.57 & 0 & 03.57 & 1.12 & 0 & 1.12 \\
\hline
\end{tabular}
\end{table}

\textbf{Discontinuous bifurcations under excitation frequency varying.} We have plotted out the oscillation amplitude dependence on excitation frequency for both system bodies that is the frequency-amplitude response. Their global view in wide range of excitation frequency is given at [10]. Now we’ll look at several partial views where discontinuous bifurcations occur.

At Fig. 5 at point B we observe phenomenon unique to non-smooth systems with discontinuous right-hand side. The point B (Fig. 6) is the point of discontinuous bifurcation. The vibroimpact system converts its motion from impactless one into motion with periodic impacts. T-periodic stable impactless regime is becoming T-periodic unstable regime with one impact per cycle – (1,1)-regime. Other regimes are arising here – stable (3,1)-regime and stable (4,2)-regime. Let us note by the way that (3,1)-periodic regime is stable in small frequency range. It is rare attractor [21].

At point B two complex conjugate Floquet multipliers \(\mu_1\) and \(\mu_2\) are leaving the unit circle. They are experiencing change by jump and accepting big values (Fig. 7). One can also see the jump of monodromy matrix in this point.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Partial view of frequency-amplitude response}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Floquet multipliers jumps under discontinuous bifurcation at frequency-amplitude response}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Set-valued Floquet multipliers jump under discontinuous bifurcation}
\end{figure}

Another partial view frequency-amplitude response in narrow range of excitation frequency is depicted at Fig. 8. At point N we observe phenomenon unique to discontinuous system – discontinuous fold bifurcation. The discontinuous
fold bifurcation connects a table branch to an unstable branch. Here set-valued Floquet multiplier $\mu$, makes huge jump along the positive real axis (Fig. 9). Its motion along positive real axis is demonstrated by Table 2.

Naturally contact impact force also has the discontinuous bifurcations at these points.

**Vibroimpact system dynamic behaviour under change of impact kind.** There is the vibroimpact system classification by different aspects [22]. One of them is impact kind characteristic – rigid or soft impact. Some classification by different aspects [22]. One of them is impact kind change of impact kind.

<table>
<thead>
<tr>
<th>$\omega_{rad}s^{-1}$</th>
<th>8.03</th>
<th>8.04</th>
<th>8.05</th>
<th>8.06</th>
<th>8.07</th>
<th>8.10</th>
<th>8.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref($\mu_1$)</td>
<td>0.595</td>
<td>0.593</td>
<td>151.4</td>
<td>90.1</td>
<td>65.9</td>
<td>37.4</td>
<td>19.7</td>
</tr>
<tr>
<td>Im($\mu_1$)</td>
<td>0.654</td>
<td>0.652</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>[1] $\mu_1$</td>
<td>0.8828</td>
<td>0.8829</td>
<td>151.4</td>
<td>90.1</td>
<td>65.9</td>
<td>37.4</td>
<td>19.7</td>
</tr>
</tbody>
</table>

![Figure 8 – Partial view of frequency-amplitude response](image1)

![Figure 9 – Floquet multiplier jump under discontinuous bifurcation](image2)

Table 2 – Floquet multipliers $\mu_1$ and $\mu_2$ jumps at point B
