UDC 539.3

V.A. BAZHENOV, O.S. POGORELOVA, T.G. POSTNIKOVA

DANGEROUS BIFURCATIONS IN 2-DOF VIBROIMPACT SYSTEM

Dynamic behaviour of strongly nonlinear non-smooth discontinuous vibroimpact system is studied. Under variation of system parameters we find the discontinuous bifurcations that are the dangerous ones. It is phenomenon unique to non-smooth systems with discontinuous right-hand side. We investigate the 2-DOF vibroimpact system by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods. We simulate the impact by nonlinear contact interactive force according to Hertz's contact law. We find the discontinuous bifurcations by Floquet multipliers values. At such points set-valued Floquet multipliers cross the unit circle by jump that is their moduli becoming more than unit by jump. We also learn the bifurcation picture change when the impact between system bodies became the soft one due the change of system parameters. This paper is the continuation of the previous works.

Keywords: Vibroimpact, Discontinuous, Hertz's law, Bifurcation, Multiplier, Nonlinear, Stability.

Introduction. Nonlinear problems are arising in many different domains of science and engineering. Often they are modeled using sets of ordinary differential equations with discontinuous right-hand side. For example they are the systems with mechanical impacts, stick-slip motion from friction, electronic switches, hybrid dynamics in control, and genetic networks [1]. Vibroimpact system is one example of such systems. Vibroimpact system is strongly nonlinear non-smooth one; the set of its motion differential equations contains the discontinuous right-hand side. Many new phenomena unique to non-smooth systems are observed under variation of system parameters. Jumps and switches in a system's state represent the grossest form of nonlinearity. Recently the investigations of such systems are developed rapidly. But today it has become clear that many aspects of dynamical behaviour of non-smooth systems aren't investigated and understood. Especially systems with impacts are of the particular interest for scientists. Under variation of system parameters a nonlinear system can often exhibit catastrophic bifurcations that destroy the desirable system state. Discontinuous bifurcations that occur in non-smooth vibroimpact systems are dangerous ones. They are hard bifurcations. Just such hard bifurcations can portend the crisis and catastrophe [2–4].

A crisis is a sudden discontinuous change in a chaotic attractor as a system parameter is varied. The crisis can be considered as a catastrophe that one endeavours to avoid. Catastrophic events can occur in different form in various kinds of nature, physics and mechanic systems. After the crisis the system state is quite different from that one before the crisis. If the nonlinear dynamical system state before the crisis is normal and desirable then the state after the crisis may be undesirable or destructive. The hard bifurcations were the subject of Catastrophe theory. Catastrophe theory was introduced in the 1960s by the renowned Field Medal mathematician Rene Thom as a part the general theory of local singularities [5]. Since then it has found applications across many areas, including biology, economics, and chemical kinetics. By investigations the phenomena of bifurcation and chaos, Catastrophe theory proved to be fundamental to the understanding of qualitative dynamics. The famous books [6, 7] are devoted to this topic. The theory was very fashionable at 70th years of 20th century. Then this fashion went away and terminology from catastrophe returned to singularities, discontinuous bifurcations and so on. But the catastrophes and crises remained. Blue Sky Catastrophes, the Swallow's Tail bifurcations are learnt by contemporary scientists [8].

The bifurcation analysis execution and the bifurcation diagrams building allow to find and to distinguish the safe, explosive, and dangerous bifurcations in dissipative dynamical systems. There are crying needs for investigations of arising of the safe, explosive and dangerous bifurcations in dynamical systems, of the crises and catastrophes for chaotic attractors. We have observed the fold catastrophe in two-body 2-DOF vibroimpact system [9–11].

We investigate the dynamic behaviour of 2-DOF vibroimpact system by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods. Short review was made in [10]. The works [12, 13] were discussed in this survey. We simulate the impact by nonlinear contact interactive force according to Hertz's contact law. Such simulation gives us the possibility to find the motion law along the whole timebase including the impact phase, to determine the impact duration and to find the contact impact forces. We find discontinuous bifurcation points where setvalued Floquet multipliers cross the unit circle by jump that is their moduli becoming more than unit by jump. It is phenomenon unique for nonsmooth systems with discontinuous right-hand side. We also learn the change of vibroimpact system dynamical behaviour when the impact between system bodies

became the soft one due the change of system parameters. This paper is the continuation of the previous works [9–11].

The aims of this paper are:

To find discontinuous bifurcations which can be the dangerous ones under variation of excitation amplitude for strongly nonlinear 2-DOF vibroimpact system.

To find discontinuous bifurcations under variation of excitation frequency.

To analyze the change of vibroimpact system bifurcation behaviour under the impact softening.

Problem formulation. The initial equations. So far as this paper is the continuation of works [9–11] the prob-

DOI: http://dx.doi.org/10.20998/2078-9130.2016.26.82732

lem formulation is the same. We'll repeat it shortly.

We analyze the dynamic behaviour of discontinuous nonlinear vibroimpact system presuming it is a two-body two-degree-of-freedom one (Fig. 1).

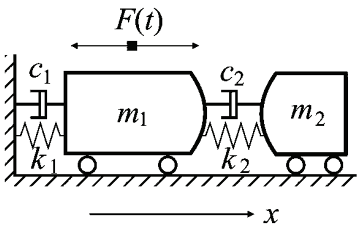

Figure 1 – Vibroimpact system model

This vibroimpact system is formed by the main body and attached one, and the latter can play the role of percussive or non-percussive dynamic damper. Bodies are connected by linear elastic springs and dampers. The main body is under the effect of periodical external force:

$$F(t) = P\cos(\omega t + \varphi_0) \quad (1)$$

We consider impacts as low velocity elastic collinear collisions without friction. The contact surfaces are smooth curvilinear ones without roughness. Thus real surface geometry in contact zone may be approximated by «Herzian» geometry.

The initial point of x coordinate is chosen in the main body mass center at the moment when all springs are not deformed. The initial distance between bodies at this moment is D. The structure of the system is experiencing transformation during the movement. The reason is its dynamic states modification forced by the impact contacts between elements.

Motion equations of the system have got the form:

$$\begin{aligned} \ddot{x}_1 &= -2\xi_1\omega_1\dot{x}_1 - \omega_1^2 x_1 - 2\xi_2\omega_2\chi(\dot{x}_1 - \dot{x}_2) - \\ &- \omega_2^2\chi(x_1 - x_2 + D) + \\ &+ \frac{1}{m_1}[F(t) - F_{con}(x_1 - x_2)], \\ \ddot{x}_2 &= -2\xi_2\omega_2(\dot{x}_2 - \dot{x}_1) - \omega_2^2(x_2 - x_1 - D) + \\ &+ \frac{1}{m_2}F_{con}(x_1 - x_2), \end{aligned} \quad (2)$$

where $\omega_1 = \sqrt{\frac{k_1}{m_1}}$, $\omega_2 = \sqrt{\frac{k_2}{m_2}}$; $\xi_1 = \frac{c_1}{2m_1\omega_1}$, $\xi_2 = \frac{c_2}{2m_2\omega_2}$; $\chi = \frac{m_2}{m_1}$, ω_1, ω_2 – partial oscillation, $F_{con}(x_1 - x_2)$ – contact interaction force, it is simulating the impact and working only during the impact. Initial conditions are:

$$x_1(0) = 0, x_2(0) = D, \dot{x}_1(0) = 0, \dot{x}_2(0) = 0. \quad (3)$$

We considered in detail the impact simulation manner in [14,15]. To simulate the impact here we use the Hertz's contact interaction force based on quasistatic Hertz's theory [16, 17]:

$$F_{con}(x_1 - x_2) = K[(x_1 - x_2)H(x_1 - x_2)]^{3/2}$$
$$K = \frac{4}{3}\frac{q}{(\delta_1 + \delta_2)\sqrt{A + B}}, \delta_1 = \frac{1 - \nu_1^2}{E_1\pi}, \delta_2 = \frac{1 - \nu_2^2}{E_2\pi}, \quad (4)$$

where $(x_1 - x_2)$ is the relative bodies rapprochement due the local deformation in contact zone, $H(x_1 - x_2)$ is the Heaviside step function, ν_i and E_i are respectively Poisson's ratios and Young's modulus for both bodies, A, B and q are the geometry characteristics of contact zone. We consider these surfaces as spherical ones, then $A = B = 1/2R_1 + 1/2R_2$, where R_1, R_2 are the contact surfaces radiuses. Only local deformations in contact zone are taken into account by the Hertz's theory. There are different proposals to make Hertz's formula more precise. Nevertheless Hertz's theory is widely used for analysis of vibroimpact system dynamics now too. Just impact simulation by nonlinear contact interaction force allows to find the motion law at all timebase including impact phase, to define impact duration and contact forces values.

Bifurcation analysis. Bifurcation analysis of vibroimpact system dynamic behaviour was fulfilled by numerical parameter continuation method in conjunction with shooting and Newton-Raphson methods [9]. Periodic motion stability or instability was determined by matrix monodromy eigenvalues that is by Floquet multipliers' values. The periodical solution is becoming unstable one if even though one Floquet multiplier leaves the unit circle in complex plane that is its modulus becoming more than unit. Such multiplier value characterizes the bifurcation kind of this bifurcation point. We have described the theoretical basis for analysis of two-body 2-DOF system in [9], numerical system parameters are given in [9–11].

Discontinuous bifurcations under excitation amplitude varying. We have plotted out the oscillation amplitude dependence on excitation amplitude for both system bodies that is the loading curves. Their global view in wide range of excitation amplitude is given at [10]. Now we'll look at their partial view where discontinuous bifurcation occurs (Fig. 2). Here and further the upper curve corresponds to attached body, the lower one – to main body. Unstable regimes are dotted by red colour. The oscillation amplitude is calculated as $A_{max} = (|x_{max}| + |x_{min}|)/2$.

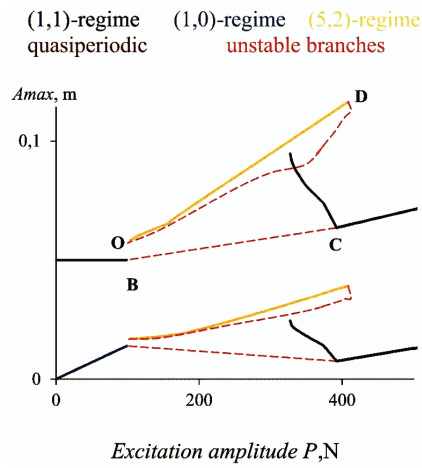

Figure 2 – Partial view of loading curves

Point B is a point of discontinuous bifurcation. It is phenomenon unique for non-smooth nonlinear system whose equations have discontinuous right-hand side. The vibroimpact system converts its motion from impactless one (section OB) into motion with periodic impacts – unstable (1,1)-regime. Other regimes – stable and unstable branches of (5,2)-regime[1] – are arising here. At point B Floquet multipliers are experiencing a discontinuous change and accepting big values [18, 19].

The set-valued Floquet multipliers cross the unit circle in direction of real positive axis by jump that is their moduli becoming more than unit by jump. Floquet multipliers behaviour in the excitation amplitude range $0 < P < 500$ N is shown at Fig. 3. Fig. 4 also shows well these jumps for multipliers μ_1 and μ_2.

Table 1 shows these jumps by numbers.

Naturally the contact force also has a discontinuous bifurcation at point B.

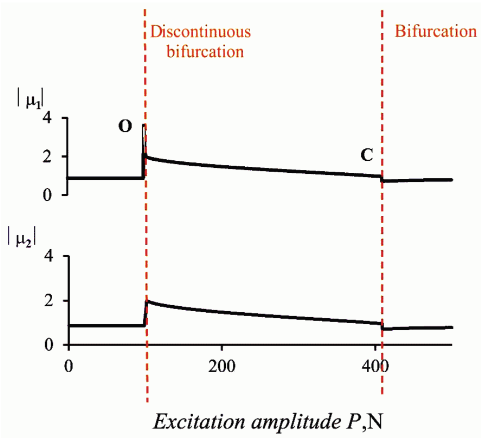

Figure 3 – Floquet multipliers jumps under discontinuous bifurcation at loading curves

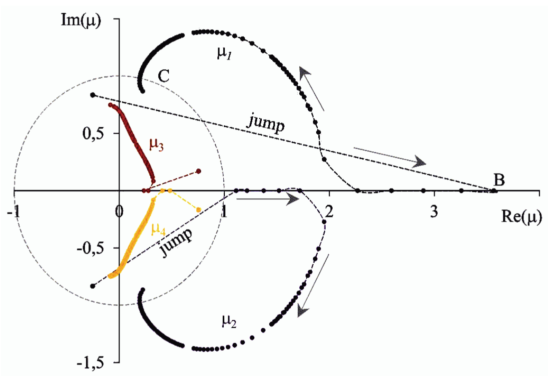

Figure 4 – Floquet multipliers behaviour in OBC section

Table 1 – Floquet multipliers μ_1 and μ_2 jumps at point B

P	Re(μ_1)	Im(μ_1)	$\lvert\mu_1\rvert$	Re(μ_2)	Im(μ_2)	$\lvert\mu_2\rvert$
98.48	-0.25	0.83	0.87	-0.25	0.83	0.87
98.98	-0.25	0.83	0.87	-0.25	0.83	0.87
99.48	3.57	0	03.57	1.12	0	1.12

Discontinuous bifurcations under excitation frequency varying. We have plotted out the oscillation amplitude dependence on excitation frequency for both system bodies that is the frequency-amplitude response. Their global view in wide range of excitation frequency is given at [10]. Now we'll look at several partial views where discontinuous bifurcations occur.

At Fig. 5 at point B we observe phenomenon unique to non-smooth systems with discontinuoius right-hand side. The point B (Fig. 6) is the point of discontinuous bifurcation. The vibroimpact system converts its motion from impactless one into motion with periodic impacts. T-periodic stable impactless regime is becoming T-periodic unstable regime with one impact per cycle – (1,1)-regime. Other regimes are arising here – stable (3,1)-regime and stable (4,2)-regime. Let us note by the way that (3,1)-periodic regime is stable in small frequency range. It is rare attractor [21].

At point B two complex conjugate Floquet multipliers μ_1 and μ_2 are leaving the unit circle. They are experiencing change by jump and accepting big values (Fig. 7). One can also see the jump of monodromy matrix in this point.

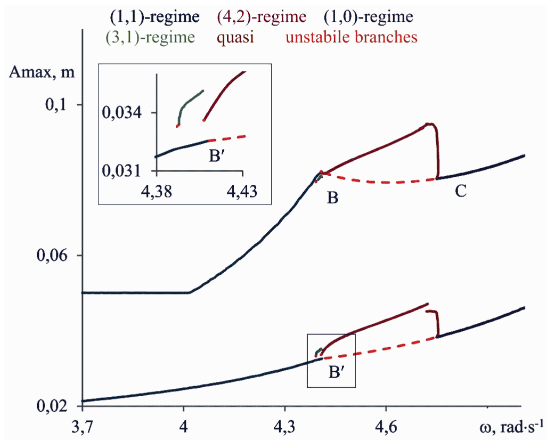

Figure 5 – Partial view of frequency-amplitude response

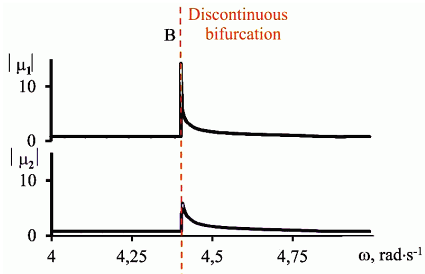

Figure 6 – Floquet multipliers jumps under discontinuous bifurcation at frequency-amplitude response

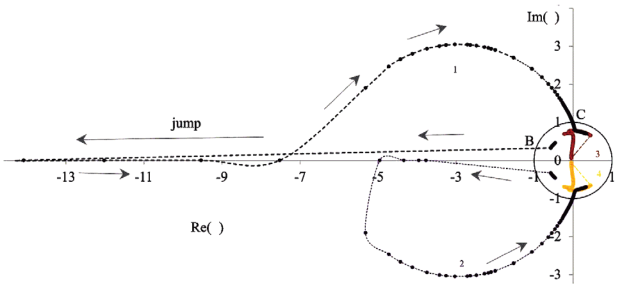

Figure7 – Set-valued Floquet multipliers jump under discontinuous bifurcation

Another partial view frequency-amplitude response in narrow range of excitation frequency is depicted at Fig. 8. At point N we observe phenomenon unique to discontinuous system – discontinuous fold bifurcation. The discontinuous

[1] We call nT -periodic regime with k impacts per cycle as (n, k)-regime [20]. T is period of external loading (1).

fold bifurcation connects a table branch to an unstable branch. Here set-valued Floquet multiplier μ_1 makes huge jump along the positive real axis (Fig. 9). Its motion along positive real axis is demonstrated by Table 2.

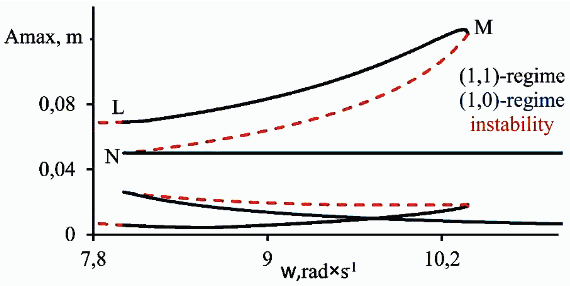

Figure 8 – Partial view of frequency-amplitude response

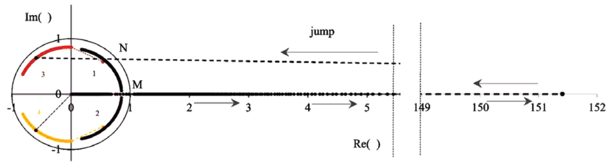

Figure 9 – Floquet multiplier jump under discontinuous bifurcation

Table 2 – Floquet multipliers μ_1 and μ_2 jumps at point B

ω_i,rad·s^{-1}	8.03	8.04	8.05	8.06	8.07	8.10	8.16
Re(μ_1)	0.595	0.593	151.4	90.1	65.9	37.4	19.7
Im(μ_1)	0.654	0.652	0	0	0	0	0
$\lvert\mu_1\rvert$	0.8828	0.8829	151.4	90.1	65.9	37.4	19.7

Naturally contact impact force also has the discontinuous bifurcation at these points.

Vibroimpact system dynamic behaviour under change of impact kind. There is the vibroimpact system classification by different aspects [22]. One of them is impact kind characteristic – rigid or soft impact. Some principal distinctions between rigid and soft contact were formulated in [23]. The main sign is its duration. Just impact duration dictates the way of its simulation. If impact duration is large then impact isn't instantaneous. Its simulation by boundary conditions with Newton's restitution coefficient using based on stereomechanic theory isn't possible [14]. The stiffness of vibroimpact system elements causes the impact softness. The soft impacts take place in engineering very often. What influence will impact kind change exert at vibroimpact system dynamic behaviour particularly at discontinuous bifurcations?

The clear criterion of impact rigidness or softness is absent. The typical trait of impact softness is its duration. Is it instantaneous or not? We have introduced the coefficient of the relative impact duration $k_{con} = T_{con}/T \cdot 100\%$ at (1,1)-regime. Here T_{con} – the time of impact that is the time of contact between bodies. Rigid impact is almost instantaneous because its duration is very small. Relative impact duration is $k_{con} = 0.09\,\%$ for the motion with rigid impact at (1,1)-regime. We have changed vibroimpact system parameters in such a way that impact between bodies became the soft one [11, 24, 25]. Relative impact duration became $k_{con} = 20.9\,\%$ for the motion with soft impact at (1,1)-regime. We have plotted both loading curves and frequency-amplitude response in wide range of excitation amplitude and frequency. We have discovered that discontinuous bifurcations are absent for our vibroimpact system with soft impact. Is it regularity or chance? Does impact softening make a vibroimpact system safer always by removing the dangerous discontinuous bifurcations? Further investigations may answer this question.

Conclusions

1. Numerical parameter continuation method provided the solution step by step and allowed to examine dynamic behaviour of two-body two-degree-of-freedom discontinuous vibroimpact system under variation of parameter continuation.

2. Impact simulation by Hertz's contact force allowed obtaining impact duration and contact forces under rigid and soft impact which wasn't instantaneous.

3. At rigid impact discontinuous bifurcations occur under variation of both excitation amplitude and excitation frequency.

4. A dangerous discontinuous fold bifurcation was observed under excitation frequency varying.

5. At soft impact discontinuous bifurcations are absent.

References: 1. *Jeffrey M. R.* Catastrophic sliding bifurcations and onset of oscillations in a superconducting resonator / *M. R. Jeffrey* et al. // Physical Review E. – 2010. – Т. 81, № 1. – С. 016213. **2.** *Grebogi C.* Critical exponent of chaotic transients in nonlinear dynamical systems / *C. Grebogi, E. Ott, J. A. Yorke* // Physical review letters. – 1986. – Т. 57, № 11. – С. 1284. **3.** *Thompson J. M. T.* Safe, explosive, and dangerous bifurcations in dissipative dynamical systems / *J. M. T. Thompson, H. B. Stewart, Y. Ueda* // Physical Review E. – 1994. – Т. 49, № 2. – С. 1019. **4.** *Фейгин М. И.* Проявление эффектов бифуркационной памяти в поведении динамической системы / *М. И. Фейгин* // Соросовский образовательный журнал. – 2001. – № 3. – С. 121-127. **5.** *Castrigiano D. P. L.* Catastrophe theory / *D. P. L. Castrigiano S. A. Hayes*. – Westview Pr, 2004. **6.** *Arnol'd V. I.* Catastrophe theory / *V. I. Arnol'd*. – Springer Science & Business Media; 2003. **7.** *Afrajmovich V. S., Il'yashenko Y. S., Shil'nikov L. P., Arnold V. I., Kazarinoff N.* Dynamical Systems V: Bifurcation Theory and Catastrophe Theory. 1994. **8.** *Shilnikov, L. P.* Showcase of blue sky catastrophes / *L. P. Shilnikov, A. L. Shilnikov, D. V. Turaev* // International Journal of Bifurcation and Chaos. – 2014. – 24(08). – С. 1440003. **9.** *Bazhenov V. A.* Stability and Bifurcations Analysis for 2-DOF Vibroimpact System by Parameter Continuation Method. Part I: Loading Curve / *P. P. Lizunov, O. S. Pogorelova, T. G. Postnikova, V. V. Otrashevskaia* // Journal of Applied Nonlinear Dynamics. – 2015. – Т. 4 (4). – С. 357–370. **10.** *Bazhenov, V. A.* Numerical Bifurcation Analysis of Discontinuous 2-DOF Vibroimpact System. Part 2: Frequency-Amplitude response / *V. A. Bazhenov, P. P. Lizunov, O. S. Pogorelova, T. G. Postnikova* // Journal of Applied Nonlinear Dynamics. –2016. – Т. 5(3). – С. 269–281. **11.** *Bazhenov V. A.* Contact impact forces at discontinuous 2-DOF vibroimpact / *V. A. Bazhenov, O. S. Pogorelova, T. G. Postnikova* // Applied Mathematics and Nonlinear Sciences. – 2016. – Т. 1, № 1. – С. 183-196. **12.** *Allgower E. L.* Introduction to numerical continuation methods / *E. L. Allgower, K. Georg*. – SIAM, 2003. – Т. 45. **13.** *Nayfeh A. H.* Applied nonlinear dynamics: analytical, computational and experimental methods / *A. H. Nayfeh, B. Balachandran*. – John Wiley & Sons; 2008. **14.** *Bazhenov V. A.* Comparison of two impact simulation methods used for nonlinear vibroimpact systems with rigid and soft impacts / *V. A. Bazhenov, O. S. Pogorelova, T. G. Postnikova* // Journal of Nonlinear Dynamics.– 2013. – № 2013:12. **15.** *Bazhenov V. A.* Comparative analysis of modeling methods for studying contact interaction in vibroimpact systems / *V. A. Bazhenov, O. S. Pogorelova, T. G. Postnikova, S. N. Goncharenko* // Strength of materials. – 2009. – № 41(4). – P. 392–398. **16.** *Goldsmith W.* Impact: the theory and physical behaviour of colliding solids / *W. Goldsmith*. – 1960. Edward Arnold, London. **17.** *Johnson K. L.* Contact mechanics, 1985 / *K. L. Johnson*. – Cambridge University Press, Cambridge; 1974. **18.** *Ivanov A. P.* Analysis of discontinuous bifurcations in nonsmooth dynamical systems / *A. P. Ivanov* // Regular and Chaotic Dynamics. – 2012. – № 17(3-4). – P. 293–306. **19.** *Leine R. I.* Bifurcations in nonlinear discontinuous systems / R. I. Leine, D. H. Van Campen, B. L. Van de Vrande // Nonlinear dynamics. – 2000. – № 23 (2). – P. 105–164. **20.** *Lamarque C. H.* Modal analysis of mechanical systems with impact non-linearities: limitations to a modal superposition / *C. H. Lamarque, O. Janin* // Journal of Sound and Vibration. – 2000. – № 235 (4). – P. 567–609. **21.** *Zakrzhevsky M.* Rare attractors in driven nonlinear systems with several degrees of freedom / *M. Zakrzhevsky, I. Schukin, V. Yevstignejev* // Transport & Engineering. – 2007. – № 24. **22.** *Blazejczyk-Okolewska B.* Classification principles of types of mechanical systems with impacts–fundamental assumptions and rules / *B. Blazejczyk-Okolewska, K. Czolczynski, T. Kapitaniak* // European Journal of Mechanics-A/Solids. 2004;23(3):517–537. **23.** *Andreaus U.* Dynamics of SDOF oscillators with hysteretic motion-limiting stop / *U. Andreaus, P. Casini* // Nonlinear Dynamics. – 2000. – № 22 (2). – P. 145–164. **24.** *Bazhenov V. A.* Change of impact kind in vibroimpact system due its parameters changing / *V. A. Bazhenov, O. S. Pogorelova, T. G. Postnikova* // MATEC Web of Conferences. – 2014. – Vol. 16. EDP Sciences. – P. 05007. **25.** *Bazhenov V. A.* Influence of system stiffness parameters at contact softness in vibroimpact system / *V. A. Bazhenov, O. S. Pogorelova, T. G. Postnikova* // Strength of Materials and Theory of Structures. – 2014. – № 92. – P. 65–77.

Bibliography (transliterated): 1. Jeffrey Mike R., et al. Catastrophic sliding bifurcations and onset of oscillations in a superconducting resonator. Physical Review E 81.1 (2010): 016213. **2.** Grebogi Celso, Edward Ott, James A. Yorke Critical exponent of chaotic transients in nonlinear dynamical systems. Physical review letters 57.11 (1986): 1284. **3.** Thompson J. M. T., Stewart H. B., Ueda Y. Safe, explosive, and dangerous bifurcations in dissipative dynamical systems. Physical Review E 49.2 (1994): 1019. **4.** Fejgin M. I. Projavlenie jeffektov bifurkacionnoj pamjati v povedenii dinamicheskoj sistemy. Sorosovskij obrazovatel'nyj zhurnal, 2001, no 3, pp. 121-127. **5.** Castrigiano D. P. L., Hayes S. A. Hayes. Catastrophe theory. Westview Pr, 2004. **6.** Arnol'd V. I. Catastrophe theory. Springer Science & Business Media, 2003. **7.** Arnol'd V. I. et al. Dynamical systems. V: bifurcation theory and catastrophe theory. Vol. 5. Springer Science & Business Media, 2013. **8.** Shilnikov, L. P., Shilnikov, A. L., Turaev, D. V. Showcase of blue sky catastrophes. International Journal of Bifurcation and Chaos 24.08 (2014): 1440003. **9.** Bazhenov V. A., et al. Stability and Bifurcations Analysis for 2-DOF Vibroimpact System by Parameter Continuation Method. Part I: Loading Curves. Journal of Applied Nonlinear Dynamics 4.4 (2015). pp. 357-370. **10.** Bazhenov V. A., et al. Numerical Bifurcation Analysis of Discontinuous 2-DOF Vibroimpact System. Part 2: Frequency-Amplitude response. Journal of Applied Nonlinear Dynamics 5.3 (2016). pp. 269-281. **11.** Bazhenov V. A., Pogorelova O. S., Postnikova T. G. Contact impact forces at discontinuous 2-DOF vibroimpact. Applied Mathematics and Nonlinear Sciences, 2016, vol. 1, no 1, pp. 183-196. **12.** Allgower E. L., Georg K. Introduction to numerical continuation methods. Vol. 45, SIAM, 2003. **13.** Nayfeh A. H., Balachandran B. Applied nonlinear dynamics: analytical, computational and experimental methods. John Wiley & Sons, 2008. **14.** Bazhenov V. A., Pogorelova O. S., Postnikova T. G. Comparison of two impact simulation methods used for nonlinear vibroimpact systems with rigid and soft impacts. Journal of Nonlinear Dynamics, 2013. **15.** Bazhenov V. A. et al. Comparative analysis of modeling methods for studying contact interaction in vibroimpact systems. Strength of materials 41.4 (2009), pp. 392-398. **16.** Goldsmith W. Impact. Courier Corporation, 2001. **17.** Johnson K. L. Contact mechanics, 1985. (1974), pp. 57-63. **18.** Ivanov A. P. Analysis of discontinuous bifurcations in nonsmooth dynamical systems. Regular and Chaotic Dynamics 17.3-4 (2012), pp. 293-306. **19.** Leine R. I., Van Campen D. H., Van de Vrande B. L. Bifurcations in nonlinear discontinuous systems. Nonlinear dynamics 23.2 (2000). pp. 105-164. **20.** Lamarque C. H., Janin O. Modal analysis of mechanical systems with impact non-linearities: limitations to a modal superposition. Journal of Sound and Vibration 235.4 (2000), pp. 567-609. **21.** Zakrzhevsky, M., Schukin, I., Yevstignejev, V. Rare attractors in driven nonlinear systems with several degrees of freedom. Transport & Engineering 24 (2007). **22.** Blazejczyk-Okolewska B., Czolczynski K., Kapitaniak T. Classification principles of types of mechanical systems with impacts–fundamental assumptions and rules. European Journal of Mechanics-A/Solids23.3 (2004), pp. 517-537. **23.** Andreaus, U., Casini, P. Dynamics of SDOF oscillators with hysteretic motion-limiting stop. Nonlinear Dynamics 22.2 (2000), pp. 145-164. **24.** Bazhenov V. A., Pogorelova O. S., Postnikova T. G. Change of impact kind in vibroimpact system due its parameters changing. MATEC Web of Conferences, vol. 16. EDP Sciences, 2014. **25.** Bazhenov, V. A., Pogorelova O. S., Postnikova T. G. Influence of system stiffness parameters at contact softness in vibroimpact system. Opir materialiv ta teoriya sporud 92 (2014), pp. 65-77.

Надійшла (received) 22.09.2016

Відомості про авторів / Сведения об авторах / About the Authors

Баженов Віктор Андрійович – доктор технічних наук, професор, академік Національної академії педагогічних наук України, директор НДІ будівельної механіки, Київський національний університет будівництва і архітектури, тел. +38(044) 245-48-29; моб.тел.: +38(067) 111-22-33; e-mail: vikabazh@ukr.net

Bazhenov V.A. – Professor, Head of Department of Structural Mechanic, KNUCA; tel.: +38(067) 111-22-33; e-mail: vikabazh@ukr.net

Погорелова Ольга Семенівна – кандидат фізико-математичних наук, старший науковий співробітник, провідний науковий співробітник НДІ будівельної механіки , Київський національний університет будівництва і архітектури, тел. +38(044) 245-48-29;мобільний тел.: +38(067) 606-03-00; e-mail: pogos13@ukr.net

Pogorelova O.S. – Candidate of Physico-mathematical Sciences, Senior Research Officer, KNUCA; tel.: +38(067) 606-03-00; e-mail: pogos13@ukr.net

Постнікова Тетяна Георгіївна – кандидат технічних наук, старший науковий співробітник, старший науковий співробітник НДІ будівельної механіки, Київський національний університет будівництва і архітектури, тел. +38(044) 245-48-29; мобільний тел.: +38(050) 353-47-19; e-mail: posttan@ukr.net

Postnikova T.G. – Candidate of Engineering Sciences, Senior Research Officer, KNUCA; tel.: +38(050) 353-47-19; e-mail: posttan@ukr.net