INFLUENCE OF STRESSES ON DEFORMATION PROCESS UNDER THE IRRADIATION CREEP AND SWELLING

Approaches for describing the deformation of structural elements made from the material, in which radiation creep and swelling strains develop simultaneously, are discussed. The technique for description of irradiation swelling strains, which is used for calculational analysis of stress-strain state arising in structural elements under the joint action of irradiation and thermal-stress fields, is regarded. A complete system of equations of the boundary –initial value problem is presented, in which elastic and thermal strains, strains of radiation creep and swelling are taken into account. Numerical modelling was carried out using the specialized software FEM Creep, in which the boundary value problem is solved by the Finite Element Method, and the initial one is integrated in time by the difference predictor-corrector method. Two forms are given for the equation of state describing the radiation swelling strains: first is for the components of the strain tensor as well as second is prepared for their rates. The hypothesis about the linear correspondence of the received radiation dose and the deformation time, during which radiation swelling strains develop, are analyzed. A number of questions that require answers when using equations with a complex stress state in which the radiation swelling strains are directly depend on stresses, are discussed. Based on the processing of experimental data on the swelling of tubes made of steel 316Ti in the temperature range of 450-460 °C, a form of the equation for the radiation swelling stress rate is proposed, and the constants included in it are determined. Using the example of numerical modelling of the deformation of tubes were made of steel 316Ti and loaded by inner pressure, the applicability of the classical approach for the analysis of the stress-strain state in the presence of radiation swelling strains is shown.

Keywords: deformation, stress, radiation creep, radiation swelling, tube.

Introduction. Determination of the residual life of the in-vessel elements of nuclear power station (NPP) reactors requires the construction of adequate constitutive equations, which reflect all the main effects that occur during the interaction of thermal, force and radiation fields [1, 2]. For a large class of reactors, which include VVER reactors installed at Ukrainian NPPs, the main processes leading to irreversible deformation of structures are radiation creep and swelling. In the range of their operating temperatures, thermal creep strains are insignificant. Also, the determining factor is the presence of thermal strains, that are inhomogeneously distributed over the volume. If today there are no special problems with the determination of the temperature stresses caused by them, as well as taking into account the influence of radiation creep, which often has a linear form of the dependence of stresses on strains, then the determination of the effect of radiation swelling on the overall picture of deformation continues to be an urgent task.

Its solution requires the presence of adequate constitutive equations, which will reflect the effect of both the direct radiation dose (or neutron fluence) and inhomogeneous temperature fields [2].

A large number of studies have been devoted to the construction of constitutive equations describing the radiation swelling of metallic materials [1, 3, 4-9]. In particular, for the class of austenitic steels, which are the...
material of a large number of in-vessel structural elements, this issue is considered in [9].

At present, two approaches are being intensively discussed, which are used to construct the governing equation for the function of radiation swelling strain. The first is the so-called “free” swelling: the process that occurs in a material under the external influence of irradiation and temperature. The functional dependence in this case includes the accumulated radiation dose or neutron fluence, temperature, time, constants describing this process. Examples of this kind of dependencies are widespread [5, 6].

Another approach involves the inclusion in the equation, in addition to the considered parameters, also of the stress function in the form of a dependence on the invariants of their tensors. The idea of such use is quite obvious - the deformation of loaded specimens does not occur in the same way as unloaded ones. The experiments carried out [3, 6] confirm this conclusion.

Further, since the studies carried out after processing the obtained data were aimed at direct construction of dependencies, the authors took into account the stress values (see, for example, [3,4,6,7]).

M. M. Hall Jr. and J. E. Flinn [4] developed a model that takes into account the different contributions of the volumetric and deviatoric components of the stress tensor on the rate of radiation creep and swelling strain. For this, the parameters included in the constitutive equations are experimentally determined. Similar models and methods for determining the constants for describing the influence of the stress tensor components are discussed in [6].

As noted above, one of the main problems arising in solving problems in which there is a decisive influence of temperature and radiation fields is the construction of adequate constitutive equations. The problem is caused, firstly, by the difficulty and low accessibility for many researchers to conduct experiments in the presence of radiation exposure. Second reason is the duration and, consequently, the high cost of the tests. In this regard, it has become a common practice to use the results published in scientific periodicals - both directly the curves describing the experimental data, and those already obtained by other authors of the equations.

It is when using the second approach the significant difficulties can meet and lead to incorrect results. This may be incomplete satisfaction of the used relations with the basic thermodynamical laws, and the absence of invariance in the transformation of coordinate systems, etc. (On this issue, see, for example, [10, 11]). The possibility of errors in the transition from the equations written for the uniaxial case to the case of a complex stress state is not excluded.

Recently, attention has been drawn to the issue of the durability of reactor internals. For the analysis of these structural elements, which have a complex geometric shape, it is necessary to solve the boundary value, and more often - the boundary - initial value problems of solid mechanics. This raises the question of taking into account the current stresses, varying over time. Traditionally, their influence is assessed indirectly, through a varying in the general level of strains caused by them [10]. An alternative approach is to directly introduce the stress function (invariants of their tensors) into the governing equation for the radiation swelling strain. These approaches are discussed in a paper. For illustration, the experimental data obtained in [3] were used.

**Foundations and numerical method.** Let us consider the main points that allow us to formulate the problem statement. For it, as usual, differential equilibria and geometric relationships are used. In connection with the further use of methods of time step-by-step discretization in solving the resolving system of differential equations, it seems acceptable to use the small strains or small strains with finite deflections approaches [10]. It is also logical to use the fundamental hypothesis about the strain additivity [10]. In the case under consideration, in addition to elastic strains, we will take into account thermoelastic ones, as well as radiation creep and swelling strains. The components of the total strain tensor $\varepsilon_{ij}$ in arbitrary time instant are determined in the following form:

$$\varepsilon_{ij} = \varepsilon_0 + \varepsilon^r_{ij} + \varepsilon^w_{ij} + \varepsilon^w_{ij},$$

(1)

where terms in right part are the strain tensor components: elastic $\varepsilon_0$, thermal $\varepsilon^r_{ij}$, radiation creep $\varepsilon^w_{ij}$ and radiation swelling $\varepsilon^w_{ij}$, $(i,j=1,2,3)$.

Thus, for quasi-static loading of a structural element under conditions of combined action of thermal force and radiation fields, the main system of resolving equations is written in the material point of a solid V:

$$\sigma_{ij} + f_i = 0, \quad \varepsilon_{ij} = \frac{1}{2}(\mathbf{u}_{ij} + \mathbf{u}_{ji} + \mathbf{u}_i \mathbf{u}_j) + \mathbf{e}_{i j} \mathbf{e}_{j i} \in \mathbf{V};$$

$$\sigma_{ij} = D_{ijkl}(\varepsilon_{kl} - \varepsilon_0^{kl} - \varepsilon^w_{kl} - \varepsilon^w_{kl}); \quad \sigma_{ij} = \tau_{ij}, \quad x_i \in \mathbf{S}^e; \quad \mathbf{u}_i \bigg|_{\mathbf{e}_{ij}} = \mathbf{u}_i,(x,0) = \varepsilon^r_{ij}(x,0) = \varepsilon^w_{ij}(x,0) = 0,$$

where $\sigma$ is stress tensor, $\mathbf{u}$ is displacement vector in material point. $n$ is unit normal vector to the volume’s boundary $\mathbf{S}$; $i=1,2,3$; $D_{ijkl}$ present the components of elastic moduli tensor; $u_i$ are the known values of displacement vector in the boundary part $\mathbf{S}_i$; $\tau_{ij}$ are known values of traction in the points of boundary part $\mathbf{S}_2$.

To formulate the governing equation for the components of the radiation swelling strain tensor, the following dependences are used:

$$\varepsilon^w_0 = \frac{1}{3} \mathbf{S}_0(D,t,\ldots) \varepsilon_{ij},$$

(3)

$$\varepsilon^w_0 = \frac{1}{3} \mathbf{S}_0(\Phi,t,\ldots) \varepsilon_{ij}.$$  

(4)

They use the function $\mathbf{S}_0$ [8, 9], or its rate [2, 12], depending on the accumulated dose $D$ or neutron fluence $\Phi$, time $t$, temperature $T$ with experimentally determined parameters.

Note that in most recent publications, dependence (3) has been used, which is directly obtained after processing the experimental results. These experiments are carried
out in a short time compared to the operating time of the in-vessel components. They determine the dependence of the radiation swelling on the radiation dose. The main hypothesis is the following. It is considered, that there is a one-to-one linear correspondence between the accumulated dose and time.

The absence of this hypothesis makes it impossible to simultaneously take into account the radiation creep (which is a function of time) and radiation swelling strains. In this regard, it is preferable to use a dependence of the type (4) and write down the main hypothesis of additivity for the strain rates or their increments

$$
\dot{\varepsilon}_y = \dot{\varepsilon}_y^T + \dot{\varepsilon}_y^{\sigma} + \dot{\varepsilon}_y^{\text{sw}},
$$

(5)

$$
d\varepsilon_y = d\varepsilon_y^T + d\varepsilon_y^{\sigma} + d\varepsilon_y^{\text{sw}},
$$

(6)

The choice of relations of the type (5) or (6) will depend on how the initial-boundary value problem is formulated - in rates or increments. The indicated reformulation of relations of the type (3) using this hypothesis is not difficult in the presence of experimental data. Note, that the methods of taking into account thermal strains, which are volumetric and caused by thermal expansion, are generally accepted [10]:

$$
\varepsilon_y^T = \alpha_y(T - T_0)\delta_y.
$$

(7)

Here $\alpha_y$ are thermal expansion coefficients of material in consideration.

Accordingly, to take into account the radiation swelling strains, which is also volumetric, relations of the type (3), (4) are used.

To formulate the equation for the radiation creep strain rate, we use the classical power Norton law [10]. Often, when processing experimental data, it turns out that the obtained dependence is linear [1, 2, 12]:

$$
\dot{\varepsilon}_y^{\text{sw}} = \frac{3}{2} B\sigma y, \quad \text{where } s_y \text{ are deviatoric components of strain tensor, } \sigma_y \text{ is von Mises equivalent stress, } B \text{ is a constant.}
$$

(8)

In conclusion, we note that it is practically impossible to analytically solve the formulated problem for real structural elements. For this, numerical methods are used, primarily FEM. The basic relationships of this method for solving the problem in consideration can be found in [13-15].

In this case, the substitution of relations of the type (3) and (8) into the main system of equations (2) also leads to the necessity of either reformulating it in rates and solving the initial problem by the difference method [14, 15], or to solving in increments of the original [2, 12].

Now we will try to consider a method based on the direct introduction of the stress function into the relations for describing radiation swelling of type (3) when solving the problem of deformation of struof elements in a complex stress state. In most publications, the authors indicate that they solve the problem using the FEM, but, unfortunately, they do not give the main relations used to formulate the problem. The type of results suggests that one of the common commercial software packages was used.

In this case, direct consideration of the radiation swelling strains is not provided in them. It is possible to implement author subroutines (user defined), but such development is also not mentioned. Thus, it is obvious that in the calculations for certain moments of time, the values of doses or fluences are set and the values of radiation swelling strains are determined. In this case, if in an equation of the type (3) there is additionally a dependence on the invariants of the stress tensor, then it is not clear what their values are used in the calculations. If the initial values of thermal stresses are taken, then the stress redistribution that occurs due to radiation creep is not taken into account. If, however, the values of stresses are used that are obtained when taking into account deformation to radiation creep, then the redistribution of stresses is only partially taken into account.

The fact is that in the process of deformation, for each moment of time, the relation represented by the second row of system (2) must be satisfied. Therefore, there is a varying of stresses in time, due, inter alia, to an increase in radiation creep and swelling strains. The absence of such an account leads to significant errors in the calculation of deformation in the case when the role of radiation swelling strains is significant.

Thus, it is possible to assume that this described above approach for computational analysis appeared as a result of attempts to use standard software systems designed to solve problems of the Solid Mechanics. Due to the impossibility of permanently taking into account the contribution of radiation swelling strains to the process of stress redistribution, the direct use of equations for free swelling gave incorrect results. The appearance of the dependences processed by the authors, in which expressions for the radiation swelling strains depending on stresses were formulated for statically definable systems, led to attempts to use them in solving initial-boundary value problems, moreover, formulated only as boundary value problems for selected time values. It is clear that such an approach requires substantial refinement based on the above considerations.

Thus, for numerical calculations of the long term stress-strain state in structural elements with a significant increase in radiation creep and swelling strains, it seems possible only to formulate the problem as an boundary-initial value (1)-(8) using equations for free swelling. It is the redistribution of stresses over time will reflect the general growth of strains, their influence on the growth of total deformation in the discussed classical formulation of the problem, of course, takes place. As shown above, it is indirected.

**Deformation of the tube loaded by internal pressure.** The considered method for solving the problem (1)-(8) is applicable to the analysis of the deformation of tubes made of austenitic stainless steel 316 Ti (stabilized with titanium). Experimental data for this steel are given in [3]. For comparison with the calculated data, the data set from this paper was chosen. In it all the necessary experimental data are presented including the strain curves.
obtained both with free swelling of tubular specimens and with their additional loading by internal pressure.

Tubes with a wall thickness of 1 mm, heated to a temperature of 450-460 °C are considered. They were irradiated to an accumulated dose of 100 dpa. Data were obtained for three series of experiments: unloaded tubes, as well as those under internal pressure, which makes hoop stresses \(\sigma_h\) in them of 90 and 170 MPa [3].

This paper presents an experimentally obtained dependence of the radiation swelling strain \(\varepsilon_{sw}^m\) % from the values of the accumulated dose, which was replaced as a time dependence. A ratio of 1.86 dpa per year was used. In fig. 1, the dots show the experimental data from [3].

![Figure 1](image)

**Fig. 1** – Comparison of experimental (dots) and numerical (line) data. Free swelling strain versus time, \(T=450 \, ^\circ\text{C}\)

An analysis of this curve shows that in the considered case, there is a nonlinear relationship between strains and time (the temperature in the considered case is constant, this made it possible to significantly simplify the relationship, highlighting the main time dependence). A power function is used for approximation.

\[
\varepsilon_{sw}^m = A t^m
\]

(9)

and

\[
\frac{d\varepsilon_{sw}^m}{dt} = \frac{1}{3} m A t^{m-1}\varepsilon_{sw}
\]

(10)

By use of experimental data the values of constants include in (9) were obtained: \(m=4.13, A=4.08 \times 10^{-2} \, \text{(h)}^{-m}\).

Further, from the plots given in [3], the constants for the equation reflecting the dependence of radiation creep on time were obtained. Initially, the experimental curves were processed based on the possible nonlinearity of the strain versus stress dependence. In this case, a value of the power of 0.95 was obtained. Further, in the calculations, the linear dependence was used

\[
\frac{d\varepsilon_{sw}^m}{dt} = B \varepsilon
\]

(11)

The value of constant: \(B=4.62 \times 10^{-10} \, \text{MPa}^{-1}\).

Further the dependencies (8), (10), (11) were used for the calculations of tubes, which were considered as 2D object at plane strain conditions.

For the computational analysis, the *FEM Creep* software package [13] was used. Due to the axial symmetry of the tubes and the loading, the pipe sector with an angle of 90° is considered.

The calculation results in comparison with the experimental data of [3] are shown in fig. 2. The values of the hoop strain \(\varepsilon_h\) % measured by the authors of this paper and obtained by calculation were compared. In fig. 2, the strains of tubes loaded with internal pressure, leading to a hoop stress in them of 170 MPa, is shown by curve 1 (calculation) and dots in the form of rhombuses; stress 90 MPa - curve 2 and triangles. Unloaded tubes are described by curve 3 and squares. To plot the graphs, the values of the components of the radiation swelling strains accumulated by the given moment were subtracted from the obtained values of the hoop strains (the experiments were carried out not in real time, but in order to obtain a given radiation dose [3]).

Let us note the qualitative agreement of the results: with an increase in the value of internal pressure, the values of strains increased.

Quantitative differences for the 2nd option (hoop stress 90 MPa) do not exceed 18%. For case 1 (hoop stress 170 MPa) at \(t = 29-40\) years, the differences between the experimental and calculated curves are significant, but further they became small. For free swelling, the differences do not exceed 10%.

Analyzing the results obtained, the conclusion that the theoretical foundations of the calculation method described in the previous section and the algorithms compiled on their basis qualitatively reflect the experimental behavior of tubes under irradiation can be done. The stress state which is added to structural elements which are deforming under irradiation conditions leads to an increase in the strain rate and large overall strain values. In this case, the calculation method includes a free swelling model. Unfortunately, the real influence of time, which is absent in the experiments of [3], introduces the described quantitative differences in the results, especially at a higher stress of 170 MPa.

![Figure 2](image)

**Fig. 2** – Comparison of experimental (dots) and numerical (line) data. Hoop strain versus time, \(T=450-460 \, ^\circ\text{C}\)

**Conclusions.** This paper presents an attempt to explain the formulation of the problem of determining the stress-strain state as an boundary- initial value problem...
using the dependence of the so-called free radiation swelling. Another approach is also discussed in which the dependence of the radiation swelling strains from stresses is used to solve the boundary value problem. Possible inconsistencies of this second approach are shown.

Using the classical formulation of the boundary - initial value problem of deformation, taking into account thermal, radiation creep and swelling strains, the problem of deformation of tubes made of austenitic stainless steel 316 Ti (stabilized with titanium) is solved. Satisfactory agreement was shown between the experimental and calculated data obtained using the FEM Creep software package [13, 14].

For a final answer to the questions posed in the paper, it is necessary to conduct a complex experiment, in which the defining contribution to the deformation process will be made by the radiation swelling and radiation creep strains. Real-time research is needed, at least for several hours, and better, of course, days with the fixation of total displacements at different points in time. Further, the experimental results should be processed using both the dose dependences of the type (3) and the time type (4). It is necessary to check the adequacy of the hypothesis about the linear correspondence of the values the radiation damage dose and time. For such a comprehensive study, it is necessary to combine the efforts of specialists in the field of radiation material science and solid mechanics.

References


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Відомості про авторів / Сведения об авторах / About the Authors

Бреславський Дмитро Васильович – доктор технічних наук, професор, завідувач кафедри комп’ютерного моделювання процесів та систем, Національний технічний університет «Харківський політехнічний інститут»; тел.: (057)-707-64-54; e-mail: brdm@kpi.kharkov.ua. ORCID: https://orcid.org/0000-0002-3792-5504

Бреславський Дмитрий Васильевич – доктор технических наук, профессор, заведующий кафедры компьютерного моделирования процессов и систем, Национальный технический университет «Харьковский политехнический институт»; тел.: (057)-707-64-54; e-mail: brdm@kpi.kharkov.ua. ORCID: https://orcid.org/0000-0002-3792-5504

Breslavsky Dmytro Vasylovych – Doctor of Technical Sciences, Professor, Head of the Department of Computer Modeling of Processes and Systems, National Technical University "Kharkiv Polytechnic Institute"; tel.: (057)-707-64-54; e-mail: brdm@kpi.kharkov.ua. ORCID: https://orcid.org/0000-0002-3792-5504