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PHYSICAL AND GEOMETRICAL NONLINEAR FORCED OSCILLATIONS OF BEAMS

The paper presents a calculation method and the results of modeling the nonlinear forced planar oscillations of a beam. The calculation approach is based on the method of weighted residuals in the Galerkin form in combination with numerical methods of integration over time. A sequential analysis of elastic linear and geometrically nonlinear oscillations is performed and the case of irreversible deformation due to the occurrence of physically nonlinear creep strains is considered. To describe it, the Norton power law is used. Cases of hinge supported and a cantilever beam are considered. When solving the problem of a hinge supported beam, the sine system was used as the basis functions, and the Krylov functions were used for the cantilever beam's problem. The results of numerical modeling are presented in the form of the dependence of the beam deflections on time and on the coordinate at a given point in time. The influence of geometric nonlinearity is demonstrated. The increase in deflection with time due to an increase in creep strains is analyzed.

Keywords: beam, geometric nonlinearity, physical nonlinearity, creep, forced oscillations, Galerkin method.

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ФІЗИЧНО ТА ГЕОМЕТРИЧНО НЕЛІНІЙНІ КОЛИВАННЯ БАЛОК

У статті представлено метод розрахунку та наведено результати моделювання нелінійних вимушених коливань балки. В основу розрахункового підходу покладено метод зважених відхилів за формою Гальоркіна у комбінації з чисельними методами інтегрування за часом. Виконано послідовний аналізпружних лінійних та геометрично нелінійних коливань та розглянуто випадок незворотного деформування, що обумовлено виникненням фізично нелінійних деформацій повзучості. Для її опису використано степеневий закон Нортону. Розглянуто випадки шарнірного закріплення та консольної балку. При розв'язанні задачі про шарнірно оперту балку як базисні функції застосовано систему синусів, у задачі про консольну балку – функції Крилова. Результати чисельного моделювання представлено у вигляді залежності прогинів балки від часу та від координати у фіксований момент часу. Проаналізовано зростання прогину у часі завдяки розвитку деформацій повзучості.

Ключові слова: балка, геометрична нелінійність, фізична нелінійність, повзучість, вимушені коливання, метод Гальоркіна.

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ФИЗИЧЕСКИ И ГЕОМЕТРИЧЕСКИ НЕЛИНЕЙНЫЕ КОЛЕБАНИЯ БАЛОК

В статье представлен метод расчета и приведены результаты моделирования нелинейных вынужденных колебаний балки. В основу подхода при расчетах положен метод взвешенных невязок в форме Галеркина в комбинации с численными методами интегрирования по времени. Выполнен последовательный анализ упругих линейных и геометрически нелинейных колебаний и рассмотрен случай необратимого деформирования, обусловленного возникновением физически нелинейных деформаций ползучести. Для ее описания использован степенной закон Нортона. Рассмотрены случаи шарнирного закрепления и консольной балки. При решении задачи о шарнирно опертой балке в качестве базисных функций использована система синусов, в задаче о консольной балке – функции Крылова. Результаты численного моделирования представлены в виде зависимости прогибов балки от времени и от координаты в заданный момент времени. Проанализирован рост прогиба со временем вследствие роста деформаций ползучести.

Ключевые слова: балка, геометрическая нелинейность, физическая нелинейность, ползучесть, вынужденные колебания, метод Галеркина.

Introduction. The cases of mixed deformation in structural elements remain as great important. The interaction of different processes can lead to appearance of new effects, a lot of them can be considered as dangerous from the point of view of strength analysis.

The process of interaction between cyclic loading and growth of irreversible strains is often met in industrial applications. That is why the great number of investigations had been done in this direction [1-2]. The main part of them were done in the field of the Theory of Plasticity. However, the long term deformation, mainly at elevated temperatures, can strongly affect the stress-strain state of a structure [3]. The case of creep strain growth at the long term cyclic loading can be analyzed by the approach, in which the method of asymptotic expansions with averaging over the period of oscillations was used [4-5]. This approach make possible to study the influence of the cyclic variation of loading on the creep process. It can be regarded as effective in the case, when the total strain, accumulated during long time, is in the consideration.

Otherwise, in many cases, especially when the load value can be regarded as big and short term creep [3] is observed, the varying of displacements and strains from one to another period can be of a great importance.

The paper presents the method of solution for regarded problem of forced oscillations which are run in the case of the creep in material. The case of beam bending is considered. Due to the above mentioned case of short term creep, the value of load must be high. In this case the beam displacements cannot be analyzed due to the linear Cauchy theory. That is why the case of geometrical nonlinear deformation is studied at first. Later the physical nonlinear process will be analyzed. The paper's results continue the method was based on the approach from [6], where simplified case without geometrical nonlinear correlations as well as one mode in Galerkin approximations were regarded.

Problem statement. Let us consider the beam with length L loaded by distributed load $f(x,t)$ along its axis (Fig. 1, 2). This loading has constant f_0 and varying part f_1, which varying due to one-harmonic sine law.

We consider the differential equation of beam's oscillations in the following form, where geometrically nonlinear term is added to the equation was presented in [6]:

$$\frac{\partial^2 u(x,t)}{\partial t^2} + a^2 \frac{\partial^4 u(x,t)}{\partial x^4} + C\frac{\partial^2 u(x,t)}{\partial x^2}\cdot$$
$$\cdot\int_0^l \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^2 dx - a^{2n}\left[\frac{\partial^4 u(x,t)}{\partial x^4}\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^{n-1} + \left(\frac{\partial^3 u(x,t)}{\partial x^3}\right)^2 (n-1)\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right)^{n-2}\right]\cdot$$
$$\cdot bt\Phi(u) = f(x,t) \qquad (1)$$

Here $u(x,t)$ are the beam's displacements which are depended from coordinate along the beam's axis x and time t; a^2 and C are the coefficients which are depended from beam's parameters. b is Norton creep law constant, creep exponent n was equal to 3. $\Phi(u)$ is the function of coordinate u which is directed normal to x in beam's cross section.

Eq.(1) was obtained in [6] by the method of direct consideration of beam's RVE. This equation is added by initial conditions

$$u(x,0) = u_0; \quad \frac{du(x,0)}{dt} = 0, \qquad (2)$$

where u_0 is static distribution of deflections at $t = 0$.

Let us analyze two cases of boundary conditions. First is hinge supported (Fig.1):

$$u(0,t) = \frac{\partial^2 u(0,t)}{\partial x^2} = u(l,t) = \frac{\partial^2 u(l,t)}{\partial x^2} = 0\,. \qquad (3)$$

As well as the second is cantilever one(Fig.2):

$$u(0,t) = \frac{\partial u(0,t)}{\partial x} = \frac{\partial^2 u(l,t)}{\partial x^2} = \frac{\partial^3 u(l,t)}{\partial x^3} = 0\,. \qquad (4)$$

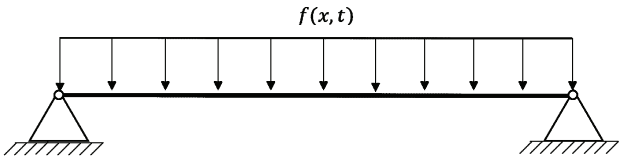

Figure 1 – Hinge supported beam

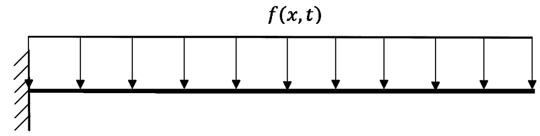

Figure 2 – Cantilever beam

Now let us consider the method of solution.

Method of solution. Due to use of combination of Galerkin's and numerical methods, the problem of results analysis and verification is important. That is why at first we consider the case of elastic oscillations in geometrically nonlinear statement, and the last term in Eq.(1) will be equal to zero. At last, zero approximation, which uses Cauchy equation,was used in order to compare results and to determine the frames of the influence of geometrical nonlinearity for considered cases.

By use of Galerkin approach [7] at $t = 0$ we have

$$\int_\Omega N_l R_\Omega d\Omega + \int_\Gamma \bar{N}_l R_\Gamma d\Gamma = 0\,. \qquad (5)$$

Here R_Ω and R_Γ are residuals, which are obtained for beam length and for beam's boundaries respectively. N_l and $\bar{N}_l$ are basis functions for beam and boundaries. They are equal to $N_i = \sin\left(\frac{(2i-1)\pi x}{l}\right)$, $i = 1,\ldots,M$ for hinge supported beam. For cantilever beam we use Krylov functions:

$$\begin{cases} N_{2k-1} = U(x) = \frac{1}{2}\left(ch\, kx - \cos kx\right), \\ N_{2k} = V(x) = \frac{1}{2}\left(sh\, kx - \sin kx\right). \end{cases}$$

$$\bar{N}_l = -a^2 N_l\,.$$

Let us present the one (l-th) row of resolving system of equations for the cantilever beam at $t = 0$ (static loading):

$$a_1\left[\int_0^l a^2 \frac{d^2 N_l}{dx^2}\frac{d^2 N_1}{dx^2}dx - a^2\frac{d^2 N_1}{dx^2}\left(N_l + \frac{dN_l}{dx}\right)_{x=l}\right] +$$
$$+a_2\left[\int_0^l a^2 \frac{d^2 N_l}{dx^2}\frac{d^2 N_2}{dx^2}dx - a^2\frac{d^2 N_2}{dx^2}\left(N_l + \frac{dN_l}{dx}\right)_{x=l}\right] =$$
$$= \int_0^l N_l f dx\,. \qquad (6)$$

Here two terms are considered in approximation. Due to use of Krylov functions, the convergence is ensured.

For hinge supported beam the row of resolving system has a form

$$a_1\int_0^l a^2 N_l \frac{d^4 N_1}{dx^4}dx + a_2\int_0^l a^2 N_l \frac{d^4 N_2}{dx^4}dx +$$
$$+a_3\int_0^l a^2 N_l \frac{d^4 N_3}{dx^4}dx = \int_0^l N_l f dx\,. \qquad (7)$$

Here a_1, a_2, a_3 are the unknowns. Three terms in Galerkin approximation are considered.

Systems (6), (7)are solved by Gauss method. After numerical experiments the influence of geometrically nonlinear term on the beam's deflection values was determined.For example, for cantilever beam such dependence is presented on Fig.3.

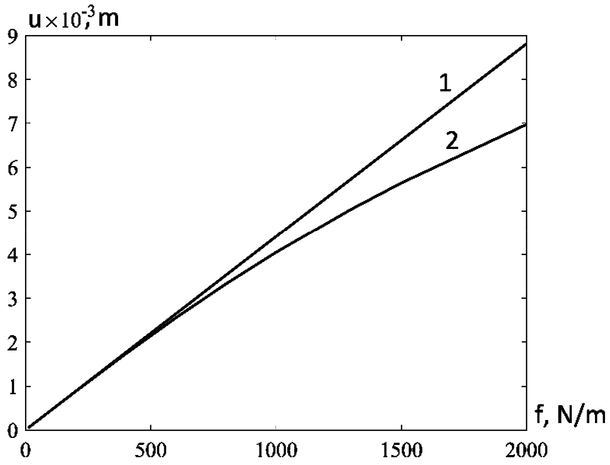

Figure 3 – Beam's deflection versus loading value

Here curve 1 demonstrates linear solution as well as curve 2 corresponds to geometrically nonlinear one. The figure shows, that the limits of the geometrically linear approach can be obtained from this comparison.

Further two methods were used in order to solve the geometrically nonlinear problem of forced oscillations of a beam. First is the Newmark method [8] and finite difference method with direct integration of equations of motions [5]. The last one is based in the finite difference approximation of the derivatives obtained in previous time step and solution the differential equation

$$a^2 \frac{\partial^4 u(x,t)}{\partial x^4} = f(x,t) - \\ -C \frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \int_0^l \left(\frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \right)^2 dx \qquad (8)$$

by iterative algorithm up to convergence ensuring.

The same approach is used and for general form of Eq.(1) and the following differential equations is solved

$$a^2 \frac{\partial^4 u(x,t)}{\partial x^4} = f(x,t) - \\ -C \frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \int_0^l \left(\frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \right)^2 dx + \\ +a^{2n} \left[\frac{\partial^4 \hat{u}(x,t)}{\partial x^4} \left(\frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \right)^{n-1} + \right. \\ \left. + \left(\frac{\partial^3 \hat{u}(x,t)}{\partial x^3} \right)^2 (n-1) \left(\frac{\partial^2 \hat{u}(x,t)}{\partial x^2} \right)^{n-2} \right] bt\Phi(u). \qquad (9)$$

in each iteration the by the Galerkin method [7].

Results of numerical simulation. Forced oscillations of hinge supported and cantilever beam.

At first let us present the comparison between results of elastic oscillations of a beam had been obtained by use of linear and nonlinear geometrical relations. The steel beam with length $L = 2$ m, cross section $S = 5 \cdot 10^{-4}$ m^2 (width $b = 0.05$ m and height $h = 0.01$ m) is considered. Physical properties: Young modulus $E = 2 \cdot 10^5$ MPa, Poisson ratio $\nu = 0.3$, mass density $\rho = 7700$ kg/m^3.

Hinge supported beam will be analyzed. It was loaded by normal distributed load

$$f = 5 + 1.5 \cdot \cos(4 \cdot \pi \cdot t) \; N/m.$$

The comparison between linear (curve 1) and nonlinear oscillations (curve 2) for the point at $x = L/2$ is presented on Fig.4.

The same analysis was made for duralumin cantilever beam with length $L = 1.3$ m and cross section $S = 1.25 \cdot 10^{-2}$ m^2 (width $b = 0.25$ m and height $h = 0.05$ m). Physical properties: Young modulus $E = 1.4 \cdot 10^5$ MPa, Poisson ratio $\nu = 0.3$, mass density $\rho = 2780$ kg/m^3.

Here the loading

$$f = 800 + 200 \cdot \cos(4 \cdot \pi \cdot t) \; N/m$$

was applied. Similar results for the right free beam's edge are presented on Fig.5.

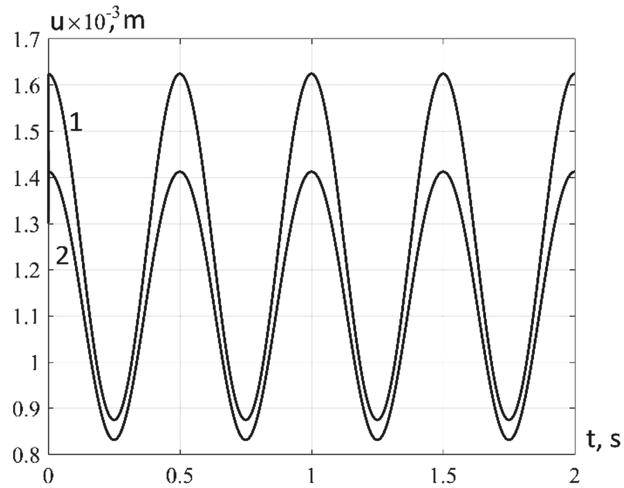

Figure 4 – Comparison between linear and nonlinear solutions for hinge supported beam

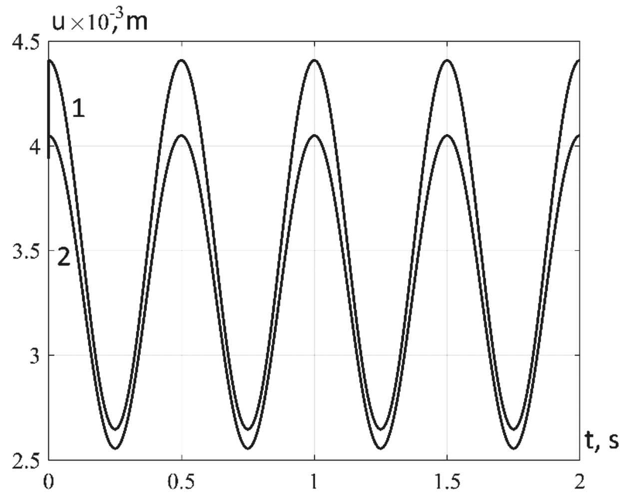

Figure 5 – Comparison between linear and nonlinear solutions for cantilever beam

Analysis of Figs 4,5 shows, that approach in use allows us to check the algorithms and confirm the well-known [9] result for deflection's reduction in the case of geometrically nonlinear description.

Further the physically nonlinear oscillations of a steel beam were studied. The steel 45X14N14V2M at $T = 600$ °C were considered, $b = 2 \cdot 10^{-10}$ (MPa)$^{-n}$/h. Let us present the numerical results for hinge supported beam with above mentioned loading. They are presented in a Table and on Figs 6,7.

The maximum deflection of a beam in different time values are presented in a Table. The points $x = 0.5 \cdot L$ and $x = 0.25 \cdot L$ were considered. The time-dependent growth of the deflection is demonstrated.

Table 1 – Deflection of a Hinge supported beam in different time moments, *m*

t, s	*x* = *L*/2	*x* = *L*/4
0	0.001624554575177	0.001158227222272
0.5	0.001412998961860	0.001008472551508
100	0.001435158775520	0.001022849293427
300	0.001487449587435	0.001056291386257
500	0.001556041599471	0.001099036162999
900	0.001916886747732	0.001305015518344

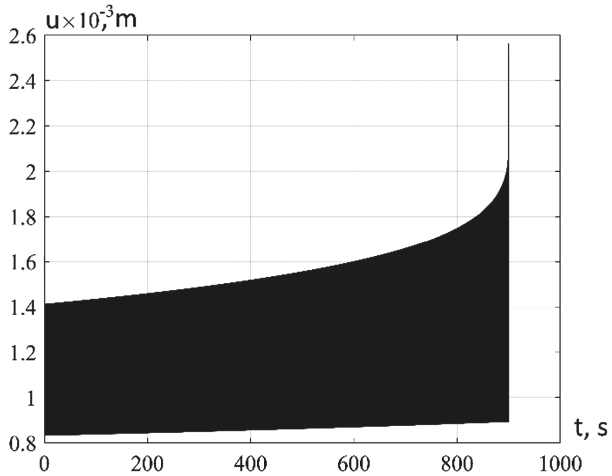

Figure 6 – Deflection at $x = 0.5$ *L* versus time

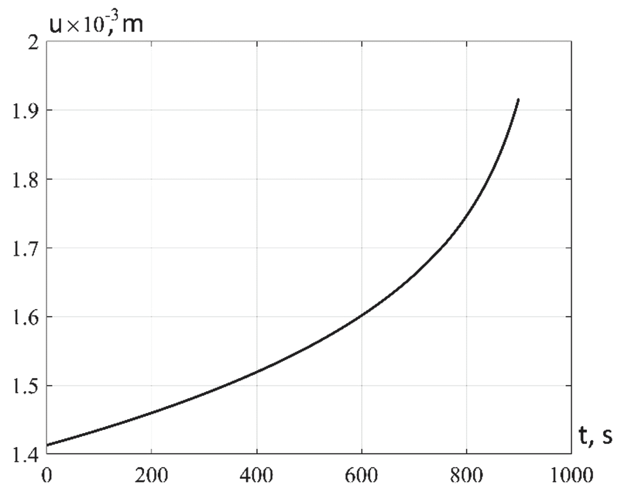

Figure 7 – Deflection envelope curve versus time

Fig. 6 presents the process of geometrically and physically nonlinear oscillations of a beam in creep conditions. The envelope curve, which is added for more clear analysis, can be found on Fig.7. After approximately 900 s the oscillations became unstable and calculations stopped. Analyzing the obtained data, one can conclude that due to creep deforming of the beam's material the irreversible growth of deflections presents.

Conclusions. The method for numerical simulation of general case of the beam's oscillations which problem statement includes geometrical and physical nonlinear creep co-relations is presented in a paper. The method includes Galerkin approximations for boundary solution as well as finite difference methods for initial one. The results show the irreversible growth of beam's deflections in creep conditions. The further problem statement has to be refined by use of damage equation in order to specify the beam behavior in last periods of oscillations.

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